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Didactic proposal for determining the density of a liquid

Propuesta didáctica para determinar la densidad de un líquido

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Abstract

This article proposes an alternative didactic methodology to determine the density of a liquid using the relationship between the mass of a body and its submerged height. An experiment is performed by placing a floating wooden cube in a transparent fish tank with water. Copper coins of equal mass are added to increase the total mass of the body, and the submerged height is measured with a tape measure. Although no direct proportionality was found between mass and submerged height according to the theoretical model, a linear relationship was observed, influenced mainly by the absorption of water in the wooden cube.

Keywords: Mass, submerged height, density, alternative methodology, theoretical model.

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Resumen

Este artículo propone una metodología didáctica alternativa para determinar la densidad de un líquido utilizando la relación entre la masa de un cuerpo y su altura sumergida. Se realiza un experimento colocando un cubo de madera flotante en una pecera transparente con agua. Se agregan monedas de cobre de igual masa para aumentar la masa total del cuerpo, y se mide la altura sumergida con una cinta métrica. Aunque no se encontró una proporcionalidad directa entre la masa y la altura sumergida según el modelo teórico, se observó una relación lineal, influenciada principalmente por la absorción de agua en el cubo de madera.

Palabras clave: Masa, altura sumergida, densidad, metodología alternativa, modelo teórico.

Introduction

Research into the factors that influence the relationship between the mass of a body and its submerged height has made significant advances in recent years. The interest in understanding the forces acting on water and how we can harness them to float without difficulty has driven numerous studies in this area.

Questions about why bodies sink and how much they sink have aroused the curiosity of scientists and have prompted extensive research. The main objective of these studies is to reveal the different factors that influence the buoyancy of bodies, including density, buoyancy, submerged volume, mass, submerged height and others.

These variables have a relationship in several sectors or areas of application, for example: at the University of Costa Rica and the National Metrology Center of Mexico (CENAM), an investigation was carried out to determine the incidence of these variables in the measurement of the density of liquids using the hydrostatic weighing method. In this study, it was possible to obtain the density of a standard solid without the need to know its volume. This methodology represents an important advance in the accurate determination of density (Hernández Sánchez, 2014) (Centeno et al., 2004). In addition, at the National Institute of Metrology Research of the Republic of Cuba (INIMET), during the calibration process of density meters, solutions with different density values are prepared. These solutions are essential to verify the accuracy of the measuring instruments used in the determination of density (Valdivia-Medina et al., 2001). (Valdivia-Medina et al., 2010).. Another study conducted at the National University of Colombia, performed density measurements of water and homeopathic medicines using a Vibrating Tube Density Meter. This approach has allowed obtaining accurate and reliable results in the determination of the density of various substances. (Pineda Garcia, 2010). In the field of agronomy, a study was carried out in the Scientific Journal UDO Agrícola where three methods were compared to determine bulk density and solidity in soils. The methods evaluated included the use of hydrometers, the Uhland method with free fall and the Uhland method with forced fall. These methods provided valuable information on the density and solidity of soils, which is essential for crop analysis and planning. (Hossne García & Cedeño Campos, 2012)..

These advances in research have contributed significantly to our understanding of the factors that influence the relationship between the mass of a body and its submerged height. More accurate measurement methods and tools have been developed, allowing us to obtain reliable results in determining the density of liquids and other materials.

Regarding didactic proposals to determine the density of different bodies, the author Fuertes (2016), describes some methods such as: hydrostatic, geometric and margin of indeterminacy, however, there is limited research regarding didactic proposals for determining the density of a liquid, this is because, currently many teachers use other types of resources for teaching science such as videos and simulations, and not promoting face-to-face experimentation. Nowadays, experimentation in the learning process is important for the student. (Fuentes, 2016).

Due to the importance of the aforementioned research, emphasis should be placed in the educational context and especially in science teaching, where these variables sometimes teachers focus more on the definition than on the structuring and deepening of a concept that demonstrates the relationship between the physical phenomenon and the variable. The latter is evident with the density variable, which is one of the most misunderstood concepts despite the fact that it is addressed from the early stages of the study of the physical sciences (Martínez-Borreguero et al., 2018)..

In the teaching of physical sciences, the problem of how to transmit knowledge to students has always arisen, for which methodologies must be structured to allow the student to develop skills from experience.

In conclusion, a remarkable process has been observed in many investigations on how to determine the density of a fluid or a solid using different methods, one more complex than others, however, for this research the didactic methodological process is promoted for student learning through experimentation, for this it is proposed to determine the density of a liquid based on the relationship the mass of the submerged body in terms of submerged looseness.

THEORETICAL FRAMEWORK

Archimedes of Syracuse mentioned "Every body immersed in a fluid experiences an upward force called thrust, equal to the weight of the fluid displaced by the body" (Falco, Franceschelli, & Marco, 2001).
We apply this principle when we place an object in water; the body sinks if its weight is greater than the weight of the displaced fluid, while the object floats when its weight is less than or equal to the weight of the displaced fluid (Falco, Franceschelli, & Marco, 2001).
Considering the experimental didactic proposal, two forces act on the object, the thrust force and the gravitational force. $(\vec{E})$ and the gravitational force $(\vec{W})$ both measured in Newtons, which in this situation have equal magnitude and opposite direction, as shown in

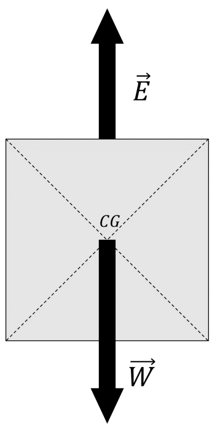

Figure 1. *Vector description of the forces acting on the object. Prepared by the author.*

Since the submerged object is in equilibrium, therefore, the sum of forces measured in Newton will be zero, as shown in equation 1. $(\sum \vec{F})$ measured in Newton will be zero, as shown in equation 1.

$$\sum \vec{F} = \vec{0} \text{ (Equation 1)}$$

Considering the positive upward reference system we have equation two:

$$\vec{E} - \vec{W} = \vec{0} \text{ (Equation 2)}$$

Where, the thrust is equal to the product of the density of the fluid (ρ) measured in grams per cubic centimeter, the acceleration of gravity (g) measured in meters per square second and the volume occupied by the object in the fluid measured in cubic centimeters. (V) measured in cubic centimeters. In the case of weight it is equal to the product of the mass of the object measured in grams and the acceleration of gravity measured in meters per second squared. (m) measured in grams and the acceleration of gravity. Therefore, replacing the above in equation 2, we have equation 3, as shown below:

$$\rho \cdot g \cdot V - mg = 0 \text{ (Equation 3)}$$

Since the object to be considered is a cubic block of wood, the volume occupied by the object in the fluid is equal to the product of the area of the base (A) measured in centimeters squared and the submerged height (h) measured in centimeters. Therefore, replacing the above in equation 3, we have equation 4, as shown below:

$$\rho \cdot g \cdot A \cdot h - mg = 0 \text{ (Equation 4)}$$

Now clearing the submerged height and simplifying the acceleration of gravity from equation 4 gives equation 5, as shown below:

$$h = \frac{m}{\rho \cdot A} \text{ (Equation 5)}$$

After having cleared the formula, it is observed that the submerged height is directly proportional to the mass of the object, therefore, if one of them increases in magnitude, the other also increases in the same proportion; which allows to obtain a theoretical model that can be verified with the experimental model obtained in practice. If we compare it with the equation of direct proportion we notice that in the vertical axis goes the submerged height, in the horizontal axis goes the mass of the body and the slope of the graph would be the inverse of the product of the density by the area. Then, our didactic experimental proposal consists of increasing the mass of the object by placing from one to seven identical coins and then observing the submerged height of the object.

Determine the relationship between the submerged height and the mass of the object including coins by means of a two-dimensional graph to calculate the density of water from its slope.

Table 1. *Experimental variables.*

Independent	Dependent	Controlled	Uncontrolled
• Mass of the object including coins	• Submerged height of the object.	• Water density • Me the coins. • Temperature of the agasas of each d • and the environment.	• Friction with water. • Water absorption in the object.

How does the submerged height of the object affect when the mass of the object is varied with coins?

In this didactic experimental proposal, the submerged height is expected to be directly proportional to the mass of the object including the coins, i.e., if the mass of the object using the coins increases, the height at which the object sinks into the fluid will increase proportionally with the same amount.

LIST OF MATERIALS AND EQUIPMENT

- A block-shaped wooden object, measuring (12, 10 ± 0.01) cm long, (8, 10 ± 0.01) cm wide and (3.50 ± 0.01) cm high with a mass of (137.00± 0.01) g
- A transparent fish tank measuring (23.00 ± 0.01) cm long, (1.40 ± 0.01) cm wide and (3.50 ± 0.01) cm high.
- Measuring tape (17.00 ± 0.01) cm
- copper coins, each with a mass of (15,00 ±0,01) g
- A balance of precision ±0.01g
- 3.75 liters of water for the fish tank.

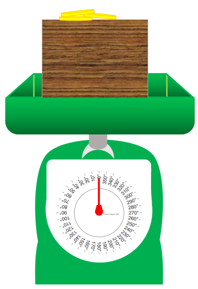

Figure 2. Measurement of the mass of the wood block including coins. Prepared by the author.

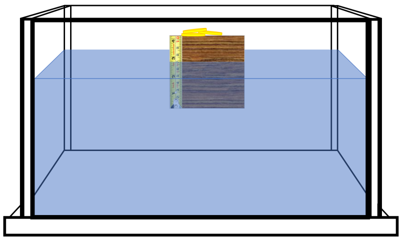

Figure 3. *Fish tank with the wooden block including the coins with the tape measure attached to take the data. Prepared by the author.*

Methodology

Initially with the wooden block, first we measure the length and width of its base to obtain its area in square centimeters, second we measure the mass in grams by means of a balance and finally we cut the tape measure and stick it on the wooden block, in such a way that the 0 of the tape measure is at the lower edge as shown in figure 3.

Now place 3.75 liters of water inside the transparent fish tank, and submerge the wooden block to obtain the submerged height measured in centimeters. Then a coin is placed on the block and its combined mass is measured again in grams, and finally the new submerged height is measured. Then another coin is added to the block of wood and the new mass is measured again to measure the submerged height, and the process is repeated until the seven coins are completed. For each mass value, 4 submerged heights will be measured from the tape measure attached to the block.

Results

The raw data obtained in the experiment are shown below.

Table 2. *Raw data of block mass including coin and their submerged heights.*

Mass of the block including coin m/g $\pm \Delta m = 0,01\ g$	Submerged height of the block h/cm $\pm \Delta h = \pm 0,01 cm$			
137, 00	1, 38	1, 40	1, 42	1, 45

152, 00	1, 60	1, 61	1, 61	1, 65
167, 00	1, 81	1, 73	1, 77	1, 69
182, 00	1, 85	1, 87	1, 86	1, 89
197, 00	1, 99	2, 02	1, 97	2, 01
212, 00	2, 11	2, 18	2, 14	2, 16
227, 00	2, 34	2, 30	2, 30	2, 31
242, 00	2, 45	2, 48	2, 39	2, 42

When performing the analysis of the submerged height measurements, it can be observed that there are groups of data for the same mass that have low accuracy and other groups of data for the same mass that have high accuracy, which is why it was decided to select from each group of masses, the 3 most accurate submerged heights. In the case of the first group of submerged heights data, we will obtain the mean and the uncertainty, as shown in equations 6 and 7 as follows:

$$\bar{h} = \frac{1{,}38 + 1{,}40 + 1{,}42}{3} = 1{,}40 \text{ cm} \text{ (Equation 6)}$$

$$\Delta h = \frac{1{,}42 - 1{,}38}{2} = 0{,}02 \text{ cm} \text{ (Equation 7)}$$

This process will be repeated with each of the groups of submerged heights found in the data table. The table of processed values is shown below:

Table 3. *Processed data of block mass including coins and average submerged height, with their respective uncertainties.*

Mass m/g $\pm\Delta m = \pm$ 0.01 g	Average height $\bar{h}$/cm	Uncertainty of average height $\pm\Delta$h/cm
137	1,40	0,02
152	1,61	0,01
167	1,73	0,04
182	1,86	0,01
197	2,01	0,02
212	2,16	0,02
227	2,30	0,01

Next, we proceed to construct the block mass graph including the coins as a function of their submerged height. The uncertainties of the measurements will be included in the graph, the line of best fit, the line of maximum and minimum slope to determine the uncertainty of the slope, as shown in Figure 4:

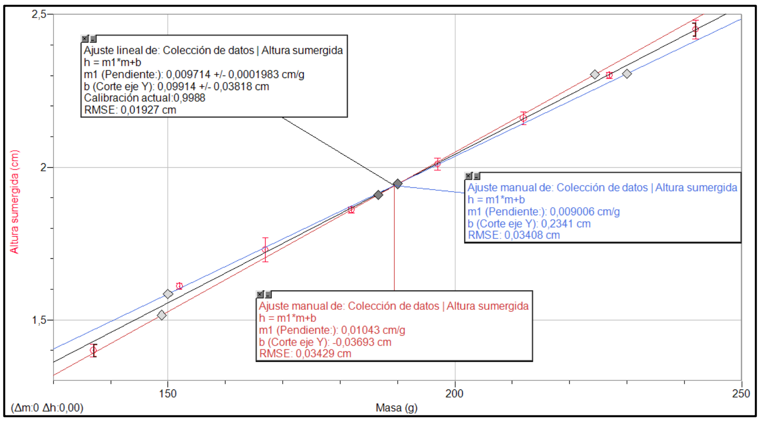

Figure 4. *Plot of the mass curve of the block including coins and average submerged height, with its uncertainties, the line of best fit, the line of maximum and minimum slope.*

From the graph it can be observed that the uncertainties of the vertical axis are too small with respect to the scale used, which is why they are not visualized in the graph. When plotting the line of best fit it can be observed that there is a linear behavior between both variables, since the line does not pass through the origin, this may be an indication of some type of systematic error. Additionally, the line of best fit passes through most of the experimental points, however, there are indications of random errors. As already mentioned in the theoretical framework, the slope (p) in Figure 4 is the inverse of the product of the submerged cross-sectional area of the wood block with the density of the water, therefore, equation 8 is obtained:

$$p = \frac{1}{\rho \cdot A} \text{ (Equation 8)}$$

From the graph we obtain the value of the slope, in this case:

$$p = 0{,}009714\ \mathrm{g}^{-1} \cdot \mathrm{cm}$$

From equation 8, we clear the density and we obtain equation 9:

$$\rho = \frac{1}{p \cdot A} \text{ (Equation 9)}$$

Now the value of slope and submerged area are replaced in equation 9 to obtain the density value:

$$\rho = \frac{1}{0{,}009714\ \mathrm{g}^{-1} \cdot \mathrm{cm}(98{,}01\mathrm{cm}^2)} = 1{,}050\ \mathrm{g} \cdot \mathrm{cm}^{-3}$$

With the help of the line of maximum and minimum slope, the uncertainty is obtained. Δpin this case the value is:

$$\Delta p = 0{,}000198\ \mathrm{gcm}^{-1}$$

Now propagating errors from equation 9 we obtain equation 10:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta p}{p} + \frac{\Delta A}{A} \text{ (Equation 10)}$$

By subtracting the uncertainty of the density from equation 10, equation 11 is obtained:

$$\Delta\rho = \rho\left(\frac{\Delta p}{p} + \frac{\Delta A}{A}\right) \text{ (Equation 10)}$$

Then replacing values in equation 10 we obtain:

$$\Delta\rho = 1{,}050 \text{ gcm}^{-3}\left(\frac{0{,}000198\ gcm^{-1}}{0{,}009714\ gcm^{-1}} + \frac{0{,}005\ cm^{-2}}{98{,}01\ cm^{-2}}\right) = 0{,}0214\text{gcm}^{-3}$$

Therefore, the density of water with its uncertainty is

$$\rho = (1{,}050\ \pm 0{,}0214)\text{gcm}^{-3}$$

Conclusions

According to the initial hypothesis and the results obtained in the graph, it can be concluded that the hypothesis was not fulfilled, since it was demonstrated that the relationship between the mass of the wood block including the coins and its height submerged in the water has a linear behavior and not a direct proportion. A possible reason why experimentally a direct proportionality between both variables was not obtained is that there is a small variation in the mass of the wood block due to the absorption of water. In our case, the wood block, having retained water, increased its mass causing the submerged height to be affected. Based on graph 1, it can be seen that the correlation coefficient (R2) is 0.9988, which means that there is a strong linear relationship between the mass of the block and its submerged height.

The value obtained for the density of water using the didactic methodological proposal was (1.050±0.021) g.cm-3, a value very close to that found in the Workshop in Hydrometer Calibration, where the density of water, under normal conditions is (0.998±0.010) g.cm-3. It can also be observed that its accuracy is low, since the percentage of error was 5.210 %, additionally the range of values obtained experimentally presents a range quite close to the range of the theoretical value, therefore, it means that the value obtained experimentally has a low accuracy. The variation of the mass of the body by absorption of water appeared as a random error that caused disformity in the measurements and a low precision at the beginning of the experiment.

The influence of readability errors in the measurements due to the refraction of light on the face of the tank is also observed, thus showing a deviation in the measurements as a systematic error of the experimentation.

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