



Development of Octave programming to calculate the forces in the structural elements of a scissor lift table

Desarrollo de la programación en Octave para calcular las fuerzas en los elementos estructurales de una mesa elevable tipo tijera

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Abstract

When building equipment or parts of a machinery, the determination of the forces acting on the elements is key to dimension them properly. Newton's laws of motion are used to determine the forces, in the case of mechanisms whose movement is slow (quasi-static) or when it does not move, the application of Newton's first law is relevant. For a table for lifting loads (weights) it is necessary to know the forces acting on the elements for different positions of the arms of the table, doing it manually requires repeated calculations and there is a possibility of placing unintentional errors when performing arithmetic operations.

Keywords: Newton of motion, program, arithmetic

Resumen

Cuando se construye un equipo o partes de una maquinaria, la determinación de las fuerzas que actúan sobre los elementos es clave para dimensionarlos adecuadamente. Para determinar las fuerzas se utilizan las leyes de Newton del movimiento, en el caso de

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mecanismos cuyo movimiento es lento (cuasi estático) o cuando no se mueve, la aplicación de la primera ley de Newton es relevante. Para una mesa elevadora de cargas (pesos) es necesario conocer las fuerzas que sobre los elementos actúan para diferentes posiciones de los brazos de la mesa, hacerlo de forma manual exige realizar los cálculos de forma repetida y hay la posibilidad de colocar errores involuntarios al hacer las operaciones aritméticas.

Palabras clave: Newton del movimiento, programa, aritmética

Introduction

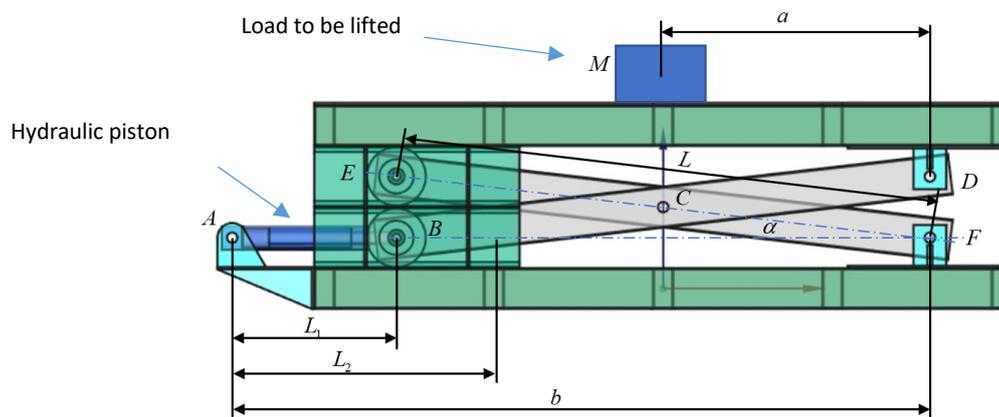
It is desired to design a scissor type lifting table with a hydraulic system that automates the upward or downward movement with a hydraulic cylinder. To determine the dimensional characteristics of the arms and the hydraulic equipment, we must obtain the numerical values of the forces acting on each member as a function of the load to be lifted and the positional angle of the arms. We try to use a computer program that allows us to calculate the forces on each member of the lifting table, and to determine the internal shear and axial forces and bending moments that allow us to size the element knowing the material with which the element will be manufactured. For this purpose we will use the help of a numerical calculation program to determine the values of forces in different positions of the table arms, from the low level to the maximum that can be raised.

For the construction of scissor-type folding tables, it is necessary to know the forces acting on the constituent elements, as well as the means for lifting the table. The study of the loads (forces) on the joints and to which the arms of the scissors mechanism are subjected is the objective of this work.

Among the means we can use for lifting the table, we will mention that these can be pneumatic, hydraulic, electric or mechanical linear actuators; For this work we select in advance that the means to lift the table will be through two hydraulic cylinders, placed at the base of the table, in horizontal position, (see figure 1) of low speed, no greater than 0.010 m/s (10 mm/s), which will allow us to analyze the mechanism practically as a static system since the inertial forces will be negligible, this is called quasi-static motion system.

To obtain the parametric formulas of the forces at each point of the folding table structure we will use the scheme shown in Figure 1, in which we introduce the relevant variables of distance and support points of the different elements.

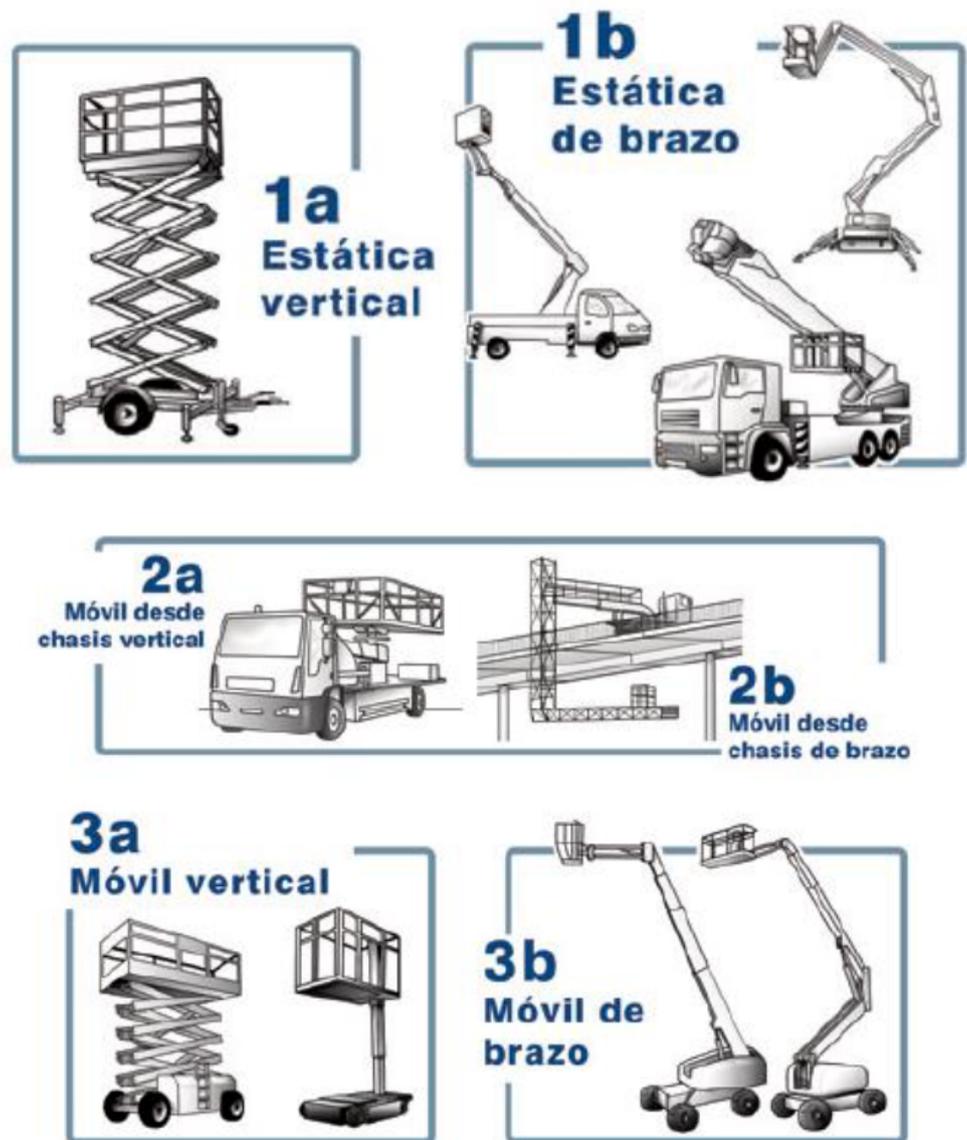
Figure 1. Simplified schematic of the scissor-type lift table, operated by a hydraulic cylinder, in its elevation view, for the low level.



One of the most widely used mechanical systems for lifting loads and people are lifting tables. These are manufactured in various arrangements or structures to meet different needs of loads or weights to be lifted. (Navarro Fandos, 2019)

In the work of Maldonado Anrubia, on the design of a scissor lift, he mentions the UNE-EN 280 classification for lifts intended for the use of people in work activities, in this he mentions the classification according to the vertical projection of the center of gravity, being called group A those whose center of gravity is always within the overturning lines of the machine and group B those that do not have their center of gravity within the overturning lines. A second complementary classification is based on the translation that the equipment can have, so we can mention type 1, in which the translation can only be performed if the equipment is in transport position; type 2, the translation of the equipment is performed when it is on a chassis; type 3 when the translation of the equipment is performed together with the platform. Figure 2 shows an example of the aforementioned classification. (Maldonado Anrubia, 2016)..

Figure 2 Types of lifting tables (Source "Instituto Nacional de Seguridad e Higiene en el Trabajo" of Spain)



Of the many possible configurations of an elevated table, the scissor type has been chosen for this work, which can be used in construction, maintenance and cleaning activities.

In the thesis of Navarro Fandos (2019), a detailed explanation is made of the constituent parts of a simple scissor type lift table, which consists of:

Base. It is the support for the rest of the constituent elements. In general it must be strong, rigid and stable, most of the times it is built to be located on the ground or inside a pit. There are also those with wheels which lock when the elevation is used.

Scissors. They provide vertical movement while supporting the platform with the load to be lifted. The scissors are connected by shafts at the center point of the arms, as well as at the top and bottom. Depending on the number of scissors the table can be single or multiple scissors with two, three, four or more pairs, depending on the height to be reached.

Platform. Dimensions can be any, but compatible with the base and scissor arms. Dimensions cannot be less than the width of the scissors when the scissors are in the lowest position.

Drive. There is a range of drives to raise and lower the platform with the scissors, they can be cable or belt mechanisms, which allow high frequencies and working speeds; with a spindle formed by nut and screw, which gives a good quality-price ratio; finally there are the lifting tables by a hydraulic mechanism that allows a long duration and smoothness of work with minimal maintenance.

It is precisely on this type of drive where the present study will be focused, for this we will mention that there are different positions in which the drive cylinder or cylinders are placed for the scissor type lift table.

Several scientific and technological papers or articles have been developed around the scissor lift tables, among which it is worth mentioning some articles in which a study of the calculation of the forces supported by the structural members of the table is made.

In Hamidi's work on the need for a dynamic analysis of certain lifting platforms for the Faculty of Mechanical Engineering of the University of Pristina, he develops his work by creating a model for the calculation of the lifting mechanism of the platform and then follows with the stresses of the structural elements by the finite element method (Hamidi, 2012).

The importance of using equation parameterization to determine forces in lifting loads on scissor lifts in a symmetrical structure is the work of Shanghong He, from the College of Mechanical and Automotive Engineering of the People's Republic of China, who conclude that the optimization of the cylinder force acting in lifting a scissor table is closely related to four position parameters of the installation and using simulation models they manage to optimize the position in which the cylinder is placed reducing by 12% the maximum cylinder load during lifting (He, Ouyang, Gong, & Liu, 2019).

The use of mathematical models has been established for scissor lift research in the work Liu & Sun. They make a kinematic and kinetic simulation analysis which was carried out with MATLAB/Simulink. The relative kinetic relationship between the hydraulic cylinder and other parts, as well as their change rules have been found. A 3D model of scissor lift mechanism was established with Pro/Engineer design software. The mechanism design was optimized in Pro/Mechanical based on the findings of simulation analysis, which can guide and improve the subsequent design. The design proved to be scientific and reasonable and could serve as a theoretical guide and reference for the scissor lift mechanism design of other uses (Liu & Sun, 2009-06).

Another study on lifting scissor tables analyzes the applicability of numerical methods using a variable in the design of lifting devices using scissor mechanisms. The study comes to the following conclusions: that the enabling dimensions of the hydraulic cylinder and the whole system can be selected on purpose and that the method proposed in the study allows them to establish force equations at the limit load locations of the system. In this way, design recommendations can be made for the dimensioning and structure of the structural elements of the scissor table (Dang & Nguyen-Dinh, 2020-11).

Methodology

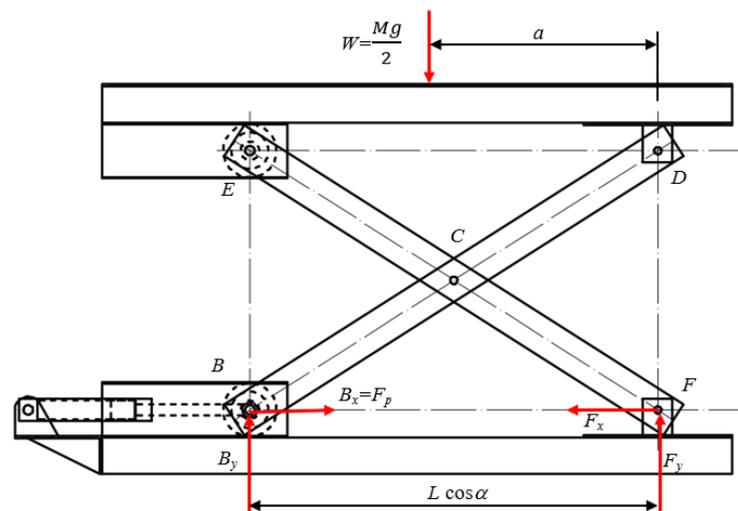
Working with parametric expressions gives us the advantage of having general expressions that can be replaced by numerical values such as different bar lengths (links), mass placement positions, position angle of the drive bar of the mechanism, etc. For the case shown in figure 1, the angle α will be the input parameter for the

formulas; the total mass M is distributed in two structures in the same way as shown in figure 1 so each structure will have half of the total weight of the assigned mass.

Determination of the parametric expression for the reactions at supports B and F.

Figure 3 shows the free body diagram of the table and the representation of the force vectors acting on the outside of the scissor lift table. It is worth mentioning that the force that will move the arms of the table making it rise or descend is represented by F_p , which is the force generated by a piston of a hydraulic system.

Figure 3. Free-body diagram (FWD) of the scissor-type lift table mechanism.



The vertical force B_y and F_y can be determined by applying Newton's first law for rigid bodies, which states that the sum of forces of bodies at rest is zero and the sum of moments. Around a point of the system, it is also zero; doing the summation of moments analysis around the point F , with the counterclockwise rotation as positive, we obtain the following expression:

$$+\odot \sum M_F = 0$$

$$aW - L \cos(\alpha) B_y = 0$$



F_y

$L \cos \alpha$

$$B_y = \frac{aW}{L \cos(\alpha)} = \frac{aMg}{2L \cos(\alpha)} \quad \text{ec. (0.1)}$$

Doing the summation of forces, we obtain:

$$+\uparrow \sum F_y = 0$$

$$B_y + F_y - W = 0 \rightarrow F_y = W - B_y$$

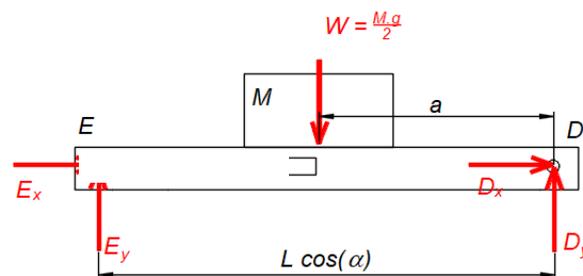
$$F_y = W - \frac{aMg}{2L \cos(\alpha)} = \frac{Mg}{2} - \frac{aMg}{2L \cos(\alpha)}$$

$$F_y = \frac{Mg}{2} \left(1 - \frac{a}{L \cos(\alpha)} \right) \quad \text{ec. (0.2)}$$

To determine the expression for the horizontal force B_x that would be the one developed by the piston of the hydraulic cylinder, we must decompose each element of the structure and place the internal forces that occur in the joints, for the effect we will analyze the forces in the table exclusively and then in the arms.

Determination of the parametric equations of the forces on supports E and D.

Figure 4. Free body diagram of the table with forces at the E and D joints.



Application of Newton's first law to rigid bodies:

$$+\odot \sum M_D = 0$$

$$aW - L \cos(\alpha) E_y = 0 \rightarrow E_y = \frac{aW}{L \cos(\alpha)}$$

$$E_y = \frac{aMg}{2L \cos(\alpha)} \quad \text{ec. (0.3)}$$

$$+\uparrow \sum F_y = 0$$

$$E_y + D_y - W = 0 \rightarrow D_y = W - E_y = \frac{Mg}{2} - \frac{aMg}{2L \cos(\alpha)}$$

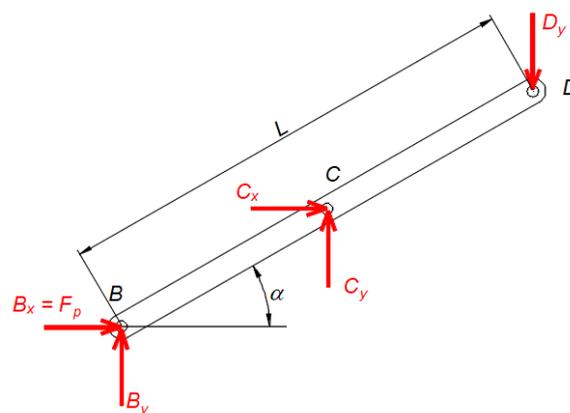
$$D_y = \frac{Mg}{2} \left(1 - \frac{a}{L \cos(\alpha)} \right) \quad \text{ec. (0.4)}$$

According to the FCD (free body diagram) there are no forces acting on the horizontal axis, it follows that E_x and D_x are zero.

Determination of the parametric equations in the BD and EF link joints.

In Figure 5, the forces on the joints constituting the link or member BD are shown. The expressions for D_y and B_y are known and B_x , C_x and C_y are unknown. In the diagram the force D_y has been drawn in the opposite direction to that seen in the free body diagram of the table, Figure 4, since Newton's third law applies when separating the elements that make up the mechanism.

Figure 5. Free body diagram of link (bar) BD. Note the change of direction of the force at joint D.



Applying Newton's first law for rigid bodies, we begin by determining the sum of moments around point C to determine the force B_x

$$\begin{aligned}
 & + \odot \sum M_C = 0 \\
 & \frac{L}{2} \sin(\alpha) B_x - \frac{L}{2} \cos(\alpha) B_y - \frac{L}{2} \cos(\alpha) D_y = 0 \\
 & \frac{L}{2} [B_x \sin(\alpha) - B_y \cos(\alpha) - D_y \cos(\alpha)] = 0 \rightarrow B_x \sin(\alpha) - B_y \cos(\alpha) - D_y \cos(\alpha) = 0 \\
 & B_x \sin(\alpha) - \frac{aMg}{2L \cos(\alpha)} \cos(\alpha) - \frac{Mg}{2} \left(1 - \frac{a}{L \cos(\alpha)}\right) \cos(\alpha) = 0 \\
 & B_x \sin(\alpha) - \frac{aMg}{2L \cos(\alpha)} \cos(\alpha) - \frac{Mg}{2} \cos(\alpha) + \frac{aMg}{2L \cos(\alpha)} \cos(\alpha) = 0 \\
 & B_x \sin(\alpha) - \frac{Mg}{2} \cos(\alpha) = 0 \\
 & B_x = \frac{Mg \cos(\alpha)}{2 \sin(\alpha)} = \frac{1}{2} Mg \cot(\alpha) \quad \text{ec. (0.5)}
 \end{aligned}$$

For the reactions at point C, we use the summation of forces on the vertical and horizontal axes.

$$\begin{aligned}
 & + \rightarrow \sum F_x = 0 \\
 & B_x + C_x = 0 \rightarrow C_x = -B_x \\
 & C_x = -\frac{1}{2} Mg \cot(\alpha) \quad \text{ec. (0.6)}
 \end{aligned}$$

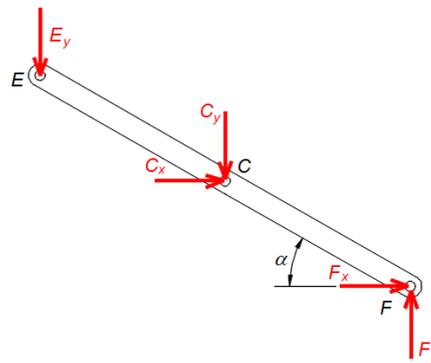
$$\begin{aligned}
 & + \uparrow \sum F_y = 0 \\
 & B_y + C_y - D_y = 0 \rightarrow C_y = D_y - B_y \\
 & C_y = \frac{Mg}{2} \left(1 - \frac{a}{L \cos(\alpha)}\right) - \frac{aMg}{2L \cos(\alpha)} = \frac{Mg}{2} - \frac{aMg}{2L \cos(\alpha)} - \frac{aMg}{2L \cos(\alpha)} \\
 & C_y = \frac{Mg}{2} - \frac{2aMg}{2L \cos(\alpha)} = \frac{Mg}{2} \left(1 - \frac{2a}{L \cos(\alpha)}\right) \quad \text{ec. (0.7)}
 \end{aligned}$$

For the link or bar EF the values can be determined since the values of the forces in C_x , C_y , E_y and F_y are known, therefore, it only remains to find the expression for the force F_x . In Figure 6 is the DCL of the link and it is worth mentioning that the direction of the forces at node C change by the application of Newton's third law, in the case

of C_x the equation 1.6 was obtained with negative sign, indicating that it points in a negative direction (to the left) of the horizontal reference of the diagram, therefore, the sign is changed for the application in the EF link.

Figure 6. Free body diagram of the EF link.

$$\begin{aligned}
 + \rightarrow \sum F_x &= 0 \\
 C_x + F_x &= 0 \rightarrow F_x = -C_x \\
 F_x &= -\frac{1}{2} Mg \cot(\alpha) \quad \text{ec. (0.8)}
 \end{aligned}$$



To quantify the variation of the forces in the joints of the lifting table mechanism, we will use the following numerical values described in Table 1. The total mass M to be lifted will be 500 kg, which will include the mass of the metallic elements of the table and the mass to be lifted. From the review of catalogs of commercial hydraulic cylinders, a cylinder of 0.400 m (400 mm) length, 0.20 m (200 mm) stroke and provisionally 50 mm bore and 30 mm rod diameter is obtained in the first instance. Therefore, the values for L_1 and L_2 will be 0.4 m and 0.6 m respectively. The value b was made numerically the same as the length of the table, which will be 1.7 m (1,700 mm). For the position where the mass M is placed on the table we assume that the center of mass coincides with half of the table length, which will be 0.65 m (650 mm).

Table 1. Numerical values of the relevant physical parameters that the elements of the folding table will have.

Parameter	Magnitude	Units	Description
M	500	kg	Block mass
L	1,3086	m	Link length
L_1	0,400	m	Piston length contracted
L_2	0,600	m	Extended piston length
a	0,650	m	Block position from end B
b	1,700	m	Position of the pivot F from end A

Results

Once all the parametric relationships of the forces on the joints have been found we can determine the operating range of the angle α . For this we know the parameters representing the length of the linear actuator when the stem is retracted, L_1 and the length of the actuator when the stem is fully out, L_2 . From this we can determine the parametric equations for the minimum and maximum value of α and thereby perform a study of the variation in joint forces.

In Figure 7 and 8, we have the two level ends that the scissor type lift table will have and will serve to determine the range of motion of the opening angle alpha (α).

Figure 7. Minimum position of the scissor lift table.

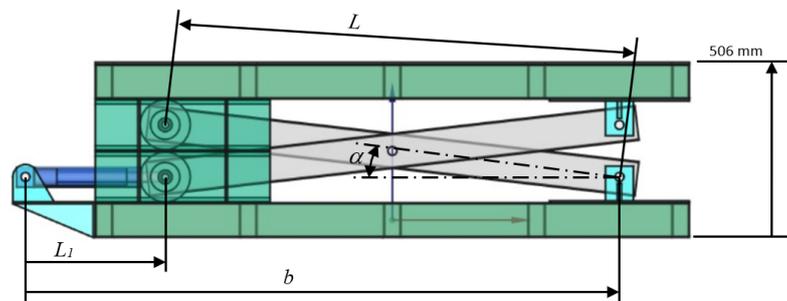
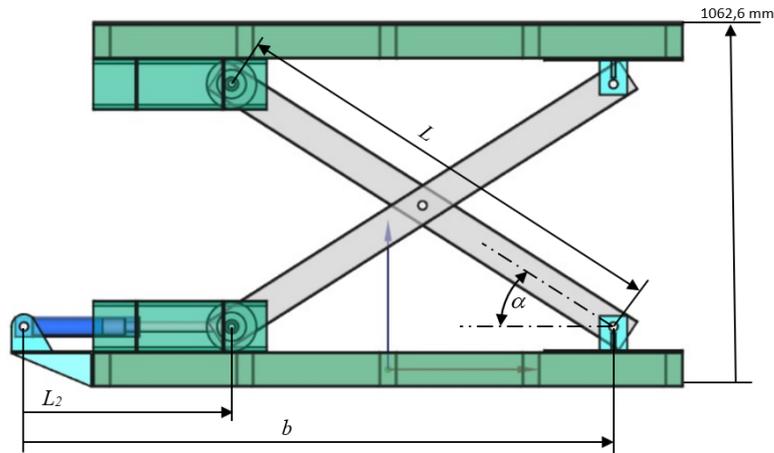


Figure 8. Maximum position of the scissor lift table.



Expression for the minimum angle: $\alpha_{MIN} = \arccos\left(\frac{b - L_1}{L}\right)$ eq. (0.9)

Expression for the maximum angle: $\alpha_{MAX} = \arccos\left(\frac{b - L_2}{L}\right)$ eq. (0.10)

Using equation 1.9 and replacing the numerical values, we obtain:

$$\alpha_{min} = \arccos\left(\frac{1\,700 - 400}{1\,308,6}\right) = 6,57^\circ \approx 6,6^\circ$$

Using equation 1.10 and replacing numerical values, we obtain:

$$\alpha_{max} = \arccos\left(\frac{1\,700 - 600}{1\,308,6}\right) = 32,79^\circ \approx 32,8^\circ$$

Therefore, for the force values obtained in this study, the opening range of the angle α will be from 6.6° to 32.7° .

To quantify the variation of the forces in the joints of the lifting table mechanism, we will make use of a computational numerical calculation program such as OCTAVE (ver. 7.1.0) with free license, which allows us to calculate equations and obtain graphs of the results. The program lines and the graphs obtained with the results of the programming of the equations with OCTAVE are presented below.

3. Development of the computer program with the free software OCTAVE, version 7.1.0.

On the program's web page, you will find that this is a scientific programming language, mathematically oriented with integrated plotting and visualization tools in 2 and 3 dimensions. The program is a free software, does not require license purchase, and is developed for different operating systems such as Linux, macOS, and Microsoft Windows.

Octave's syntax is compatible with that of another mathematical program such as Matlab, which is a proprietary program. Information on the use of Octave can be obtained from the product's web site, https://wiki.octave.org/Using_Octave

We are going to detail the instruction lines of the script that has been developed to determine the main reactions in the elements of the scissor mechanism of the lifting table.

The first lines indicate the title and letter assignment for the mechanism variables, as shown in Figure 1.

```
1 % Calculation of forces on elevated platform elements
2 clear % Command to clear previously created variables
3 clc % Command for clearing write window (terminal)
4 M = 500; % Mass of the body to be lifted in kg.
5 L = 1.3086; % Length of table arms in m.
6 L1 = 0.400; % Length of hydraulic cylinder, retracted, in m.
7 L2 = 0.600; % Length of the hydraulic cylinder completely outside,
in m.
8 a = 0.650; % position of the mass from the right edge of the table,
in m.
9 b = 1,700; % Length from cylinder base to joint F, in m.
10 g = 9.81; % Gravity in m/s^2.
11 %
```

Then we write the equations to determine the angle α of opening of the table arms, both minimum and maximum, and the number of

angular divisions with which we will calculate the forces, for this case it will be 100 angular divisions between α min and α max.

12 $\alpha_1 = \arccos((b-L_1)/L)$; % minimum angle, position below the table

13 $\alpha_2 = \arccos((b-L_2)/L)$; % max. angle, high table position

14 $n_parts = (\alpha_2 - \alpha_1)/100$; % division of the angular path in 100 parts

15 $\alpha = (\alpha_1:n_parts:\alpha_2)$;

Having done so, we write the parametric equations of the forces at joints B, F, C, E, and D, both for the horizontal and vertical components, as well as their resultant.

16 % Parametric equations of forces in the joints

17 $B_y = 0.5 * M * g * a ./ (L * \cos(\alpha))$; % Normal force at joint B, in N

18 $F_p = 0.5 * M * g * \cot(\alpha)$; % Piston force at joint B, in N.

19 %

20 $C_x = -F_p$; % Force at joint C, horizontal x-axis.

21 $C_y = 0.5 * M * g * (1 - 2 * a ./ (L * \cos(\alpha)))$; % Force at C-joint, vertical axis

22 $C = \sqrt{C_x.^2 + C_y.^2}$; % Resultant of the force in C

23 %

24 $F_x = C_x$; % Force at joint F, horizontal axis.

25 $F_y = 0.5 * M * g * (1 - a ./ (L * \cos(\alpha)))$; % Force on joint F, vertical axis

26 $F = \sqrt{F_x.^2 + F_y.^2}$; % Resultant of the force in F

27 %

28 $D_y = F_y$; % Force at table support D

29 $E_y = B_y$; % Force at table support D

30 %

The following instructions plot the database with the results of the 100 values calculated for each parametric variable.

```
31 alpha_degree = alpha*180/pi; % conversion from radians to  
sexagesimal degrees
```

```
32 %
```

```
33 figure(1)
```

```
34 plot(alpha_degree,Fp);
```

```
35 title('Horizontal force at joint B')
```

```
36 xlabel('Angle in degrees')
```

```
37 ylabel('Force in N')
```

```
38 grid
```

```
39 figure(2)
```

```
40 plot(alpha_grade,By,'r')
```

```
41 title('Vertical force at joint B')
```

```
42 xlabel('Angle in degrees')
```

```
43 ylabel('Force in N')
```

```
44 grid
```

```
45 figure(3)
```

```
46 plot(alpha_degree,Cx);
```

```
47 title('Horizontal force at the C-joint')
```

```
48 xlabel('Angle in degrees')
```

```
49 ylabel('Forces in N')
```

```
50 grid
```

```
51 figure(4)
```

```
52 plot(alpha_degree,Cy);
```

```
53 title('Vertical force at the C-joint')
54 xlabel('Angle in degrees')
55 ylabel('Forces in N')
56 grid
57 figure(5)
58 plot(alpha_degree,Ey,alpha_degree,Dy);
59 title('Forces in E and D joints');
60 xlabel('Angle in degrees');
61 ylabel('Forces in N')
62 legend('Ey','Dy');
63 grid
64 figure(6)
65 plot(alpha_degree,Fx);
66 title('Horizontal force at joint F');
67 xlabel('Angle in degrees');
68 ylabel('Forces in N')
69 grid
70 figure(7)
71 plot(alpha_degree,Fy);
72 title('Vertical force at joint F');
73 xlabel('Angle in degrees');
74 ylabel('Forces in N')
75 grid
76 %
```

Change of axes for the forces in the BD link.

For the study of the loads and stresses to which the BD link is subjected, the axes on which the forces are determined must be changed. During the development of the equations, the conventional x (horizontal) and y (vertical) axes were used, but it is convenient to change the axes to a normal or axial axis (parallel to the axis of the link) and another transverse or tangential axis (perpendicular to the axis of the link). To perform the change of axes, the equations formed through trigonometric relations will be taken, as shown in the attached figure.

The equations that allow the change can be written in an ordered form obtaining a system of equations of the following form:

$$\begin{aligned} F_n &= F_x \cos(\alpha) + F_y \sin(\alpha) \\ F_t &= -F_x \sin(\alpha) + F_y \cos(\alpha) \end{aligned} \quad \text{ec. (0.11)}$$

If we use the matrix form to write the equations, we would have the following expression:

$$\begin{bmatrix} F_n \\ F_t \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad \text{ec. (0.12)}$$

The 2 x 2 matrix containing the trigonometric expressions is called the rotation matrix and can be used to build a computer program for numerical calculation of the axial and transverse components of the horizontal and vertical forces of the link. We will again use the OCTAVE program to generate the matrix equations and plot the variation of the forces, but in the normal and transverse axes of the BD link.

The graph of link BD is shown in Figure 14, with its horizontal, x-axis, and vertical, y-axis, components, with its forces taken to the normal, n, and transverse, t, axes.

The axis transformation equations for joints B, C and D would be:

$$\begin{bmatrix} B_n \\ B_t \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} B_x \\ B_y \end{bmatrix} \quad \text{ec. (0.13)}$$

$$\begin{bmatrix} C_n \\ C_t \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} C_x \\ C_y \end{bmatrix} \text{ ec. (0.14)}$$

$$\begin{bmatrix} D_n \\ D_t \end{bmatrix} = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} 0 \\ -D_y \end{bmatrix} \text{ ec. (0.15)}$$

Next, the programming lines as a continuation of the previous lines, to obtain normal and transverse forces and the graphs obtained in the joints:

```
77 % Calculation of normal and axial forces on link BD
78 n = length(alpha);
79 forcesB = [Fp;By];
80 forcesC = [Cx;Cy];
81 array_zeros = zeros(1,n);
82 forcesD = [array_zero;-Dy];
83 forceBrot = zeros(2,n);
84 forceCrot = zeros(2,n);
85 forceDrot = zeros(2,n);
86 % loop to determine the normal and axial vectors of link BD
87 for i = 1:n
88     rotation_matrix = [cos(alpha(i)) sin(alpha(i));-sin(alpha(i))
cos(alpha(i))];
89     forceBrot(:,i) = rotation_matrix*forceB(:,i);
90 forceCrot (:,i) = rotation_matrix*forceC(:,i);
91     forceDrot(:,i) = rotation_matrix*forceD(:,i);
92 endfor
93 %
94 figure(8)
```

```
95 plot(alpha_degree,forceBrot(1,:));
96 title('Normal forces at joint B')
97 xlabel('Angle in degrees')
98 ylabel('Force in N')
99 grid
100 figure(9)
101 plot(alpha_degree,forceBrot(2,:),'r');
102 title('Transverse forces at joint B')
103 xlabel('Angle in degrees')
104 ylabel('Force in N')
105 grid
106 figure(10)
107 plot(alpha_degree,forceCrot(1,:));
108 title('Normal forces in the C-joint')
109 xlabel('Angle in degrees')
110 ylabel('Force in N')
111 grid
112 figure(11)
113 plot(alpha_degree,forceCrot(2,:),'r');
114 title('Transverse forces in the C-joint')
115 xlabel('Angle in degrees')
116 ylabel('Force in N')
117 grid
118 figure(12)
119 plot(alpha_degree,forceDrot(1,:));
```

```
120 hold on
121 plot(alpha_degree,forceDrot(2,:),'r');
122 title('Normal and transverse forces at joint D')
123 xlabel('Angle in degrees')
124 ylabel('Force in N')
125 legend('Dn','Dt');
126 grid
127 %
```

Determination of the diagram of axial forces, shear forces and bending moment in the BD link (bar) of the lifting table.

To make the graphs with the shear forces and bending moments of bar BD we will determine three positions of the table whose angles will be 6.6° , 20° and 32.7° ; at these positions we will calculate the values of normal and transverse forces at nodes B, C and D, with the help of the program developed with the free software OCTAVE for numerical calculations. The results obtained also apply to link EF since it presents the same numerical values of forces.

```
1 % Calculation of forces on elevated platform elements
2 clear
3 clc
4 % Data
5 M = 500; % Mass of the body to be lifted in kg.
6 L = 1.3086; % Length of the bars in m.
7 L1 = 0.400; % Piston rod length retracted, m.
8 L2 = 0.600; % Piston rod length fully out, in m.
9 a = 0.650; % Length of the position from the right edge of the mass,
in m.
10 b = 1.700; % Length from cylinder base to joint F, in m.
```

```
11 g = 9.81; % Gravity in m/s^2.
12 %
13 alpha = [6.6*pi/180 20*pi/180 32.7*pi/180]; % conversion of
angles in radians
14 %
15 By = 0.5*M*g*a./(L*cos(alpha)); % vertical component at joint B in
N
16 Fp = 0.5*M*g*cot(alpha); % Piston force at joint B in N.
17 Cx = -Fp;
18 Cy = 0.5*M*g*(1-2*a./(L*cos(alpha)));
19 C = sqrt(Cx.^2+Cy.^2);
20 Fx = Cx;
21 Fy = 0.5*M*g*(1-a./(L*cos(alpha)));
22 F=sqrt(Fx.^2+Fy.^2);
23 Dy = Fy;
24 Ey = By;
25 %% Calculation of normal and transverse forces of the BD link
26 forcesB = [Fp;By];
27 forcesC = [Cx;Cy];
28 array_zeros = zeros(1,3);
29 forcesD = [array_zero;-Dy];
30 forceBrot = zeros(2,3);
31 forceCrot = zeros(2,3);
32 forceDrot = zeros(2,3);
33 % loop to determine the normal and transverse vectors of link BD
```

```
34 for i = 1:3
35     rotation_matrix = [cos(alpha(i))  sin(alpha(i));-sin(alpha(i))
cos(alpha(i))];
36 forceBrot(:,i) = rotation_matrix*forceB(:,i);
37 forceCrot(:,i) = rotation_matrix*forceC(:,i);
38 forceDrot(:,i) = rotation_matrix*forceD(:,i);
39 endfor
40 Bn = forceBrot(1,:);
41 Bt = forceBrot(2,:);
42 Cn = forceCrot(1,:);
43 Ct = forceCrot(2,:);
44 Dn = forceDrot(1,:);
45 Dt = forceDrot(2,:);
46 %
```

Results displayed in the OCTAVE command window:

```
Bn =
    2.1197e+04    6.7752e+03    3.9968e+03
Bt =
   -1218.06   -1086.40   -845.61
Cn =
   -2.1056e+04   -6.3798e+03   -3.4539e+03
Ct =
    2436.1    2172.8    1691.2
Dn =
   -140.93   -395.42   -542.87
Dt =
   -1218.06   -1086.40   -845.61
```

From the results obtained from the program we made a table to sort the values appropriately.

To establish the parametric equations that allow us to obtain the diagrams of shear forces and bending moments in the bar BD, we will analyze the bar disregarding the self-weight and rotating the normal and transverse axes so that these are aligned with the horizontal and vertical axes to facilitate the determination of the equations. In the diagram of the free body diagram all the forces have been drawn in positive direction, according to the table the values of C_t are positive but the values of the forces B_t and D_t are negative therefore these forces point downwards.

To find a mathematical expression for B_t and D_t as a function of C_t , we work with summation of moments around one of the extremes, if this is D, we will have:

$$\begin{aligned}
 + \odot \sum M_D &= 0 \\
 -LB_t - (L-r)C_t &= 0 \rightarrow LB_t = -(L-r)C_t \\
 B_t &= -\left(\frac{L-r}{L}\right)C_t \quad \text{ec. (0.16)}
 \end{aligned}$$

For an angular position α to obtain the shear and bending moment diagrams, we will work with the method of sections to develop the parametric formulas.

Section BC:

$$\begin{aligned}
 + \uparrow \sum F_y &= 0 \\
 B_t - V &= 0 \rightarrow V = B_t = -\left(\frac{L-r}{L}\right)C_t \\
 V &= -\left(1 - \frac{r}{L}\right)C_t \quad \text{ec. (0.17)}
 \end{aligned}$$

$$\begin{aligned}
 + \odot \sum M_o &= 0 \\
 -xB_t + M_f &= 0 \\
 M_f &= xB_t \\
 M_f &= -x\left(1 - \frac{r}{L}\right)C_t \quad \text{ec. (0.18)}
 \end{aligned}$$

Note: since the vertical force at point C is in the middle of the bar length, the force is equally distributed at points B and D, the parametric expression can be used in general when point C is not in the middle of the bar.

CD Section:

$$\begin{aligned}
 + \uparrow \sum F_y &= 0 \\
 B_t + C_t - V &= 0 \rightarrow V = B_t + C_t \\
 V &= -\left(1 - \frac{r}{L}\right)C_t + C_t = C_t \left[-\cancel{x} + \frac{r}{L} + \cancel{x}\right] \\
 V &= \frac{r}{L}C_t \quad \text{ec. (0.19)}
 \end{aligned}$$

$$\begin{aligned}
 + \odot \sum M_o &= 0 \\
 -xB_t - (x-r)C_t + M_f &= 0 \\
 x\left(1 - \frac{r}{L}\right)C_t - (x-r)C_t + M_f &= 0 \\
 \cancel{x}C_t - \frac{r}{L}xC_t - \cancel{x}C_t + rC_t + M_f &= 0 \\
 rC_t\left(1 - \frac{x}{L}\right) + M_f &= 0 \\
 M_f &= -rC_t\left(1 - \frac{x}{L}\right) \quad \text{ec. (0.20)}
 \end{aligned}$$

Summarizing, the parametric equations of the BD member would be as follows:

$$\left. \begin{aligned}
 V &= -\left(1 - \frac{r}{L}\right)C_t \\
 M_f &= -x\left(1 - \frac{r}{L}\right)C_t
 \end{aligned} \right\} 0 \leq x < r$$

$$\left. \begin{aligned} V &= \frac{r}{L} C_t \\ M_f &= -r C_t \left(1 - \frac{x}{L} \right) \end{aligned} \right\} r \leq x \leq L$$

Using the free OCTAVE software to perform the programming, we will obtain the shear force and bending moment plots on the member. Below are the program lines and the obtained plots:

```
1 % Diagram of shear force and bending moment in link BD
2 % for the hydraulic folding table.
3 clear
4 clc
5 %Data:
6 Ct = 2436.1; % Ct force value in newtons
7 L = 1.4086; % Length of bar BD in meters
8 r = 0.7043; % Length of force application Ct in meters
9 %
10 % Shear and bending moment equations
11 x = linspace(0,L,2000);
12 for i = 1:2000
13 if x(i)<=r
14 V(i) = -(1-r/L)*Ct;
15 M(i) = -(1-r/L)*Ct*x(i);
16 else
17 V(i) = r/L*Ct;
18 M(i) = -r*Ct*(1-x(i)/L);
19 endif
```

```
20 endfor
21 % graphing of shear forces and bending moments versus x
22 figure(1)
23 plot(x,V)
24 title('Shear force in link BD')
25 xlabel('x in meters')
26 ylabel('Shear in N')
27 axis([0,1.4086,-1500,1500])
28 grid
29 figure(2)
30 plot(x,M)
31 title('Bending moment at link BD')
32 xlabel('x in meters')
33 ylabel('Moment in N.m')
34 axis([0,1.4086,-1000,0])
35 grid
36 %
37 V_max = max(abs(V))
38 [M_max,pos] = max(abs(M));
39 x_max = L*(pos-1)/2000, M_max = M_max
40 %
```

Then, the maximum values of shear force and bending moment, which are displayed in the software command window:

```
V_max = 1218.0
x_max = 0.7036
M_max = 857.44
>>
```

From the internet sites <https://matmatch.com/es/materials/minfc37594-sae-j403-grade-1008>, and <https://acerosluchriher.com/acero-sae-1008-y-1010/>, mechanical properties for SAE J 403 1008 steel are obtained, which are summarized in Table 3.

Table 2. Some physical and mechanical properties of the SAE J 403 1008 steel of the structural rectangular pipes of the DIPAC company, located in Manta-Ecuador.

Property	Value	Unit
Density	7 800 a 7 900	kg/m ³
Elastic modulus	200 a 215	GPa
Shear modulus	82	GPa
Poisson's ratio	0,29	
Yield Limit	286	MPa
Maximum tensile stress	338	MPa
Maximum elongation	20	%
Area reduction	45	%
Hardness	56	HRB

Since the mechanism will be moving in different positions, the loads will be fluctuating, therefore a conservative value of the factor of safety for the allowable stress of the material will be used (Mott, 2009). (Mott, 2009).

$$\sigma_{permissible} = \frac{\sigma_{cedencia}}{8} \quad \text{ec. (0.21)}$$

Replacing values:
$$\sigma_{permissible} = \frac{286}{8} = 35,8 \text{ MPa}$$

Applying eq. 1.22 and clearing the section modulus:

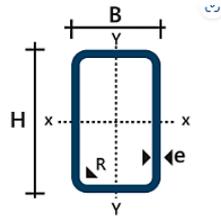
$$S = \frac{M_f}{\sigma_{permissible}} = \frac{857\,440 \text{ N}\cdot\text{mm}}{35,75 \frac{\text{N}}{\text{mm}^2}} = 23\,984,3 \text{ mm}^3 \approx 24,0 \text{ cm}^3$$

From the firm Metal Hierro, the rectangular tube has the following characteristics:

TUBO RECTANGULAR NEGRO PESADO

Los tubos estructurales rectangular hierro presentan mejoras significativas que redundan en beneficio del usuario, tales como el ahorro de soldaduras para hacer cajas, facilidad de instalación y ahorros significativos en tiempo. La opción del tubo estructural obedece a la necesidad de brindarle al cliente una mayor capacidad estructural con el mejor acabado para embellecer la construcción.

Norma:	NTE INEN 2415
Acabados:	Acero negro o Galvanizado
Largo Normal:	6.00 m
Dimensiones en Stock:	Desde 40mm x 80mm a 100mm x 200mm
Espesores:	Desde 1,50 a 6,00 mm
Observaciones:	Otras dimensiones y espesores, previa consulta



Tubería Estructural Rectangular										
Designaciones		Área	Peso	Propiedades Estáticas						
				Eje x - x			Eje y - y			
				Momento de inercia	Módulo de resistencia	Radio de giro	Momento de inercia	Módulo de resistencia	Radio de giro	
B	H	e	A	P	I	W	i	I	W	i
mm	mm	mm	cm ²	kg/m	cm ⁴	cm ³	cm	cm ⁴	cm ³	cm
40	80	1,50	3,45	2,71	28,99	7,25	2,90	9,94	4,97	1,70
		1,80	4,11	3,22	34,09	8,52	2,88	11,64	5,82	1,68
		2,00	4,54	3,56	37,36	9,34	2,87	12,73	6,36	1,67
		3,00	6,61	5,19	52,28	13,07	2,81	17,59	8,79	1,63
		4,00	8,55	6,71	64,90	16,22	2,76	21,59	10,79	1,59
50	100	2,00	5,74	4,50	74,99	15,00	3,62	25,68	10,27	2,12
		3,00	8,41	6,60	106,49	21,30	3,56	36,09	14,44	2,07
		4,00	10,95	8,59	134,24	26,85	3,50	45,05	18,02	2,03
50	150	2,00	7,74	6,07	207,54	27,67	5,18	37,21	14,88	2,19
		3,00	11,41	8,96	298,58	39,81	5,12	52,68	21,07	2,15
		4,00	14,95	11,73	381,49	50,87	5,05	66,27	26,51	2,11
100	150	3,00	14,41	11,31	460,67	61,42	5,65	247,67	49,53	4,15
		4,00	18,95	14,67	594,71	79,29	5,60	318,67	63,73	4,10
		5,00	23,36	18,33	719,46	95,93	5,55	384,27	76,85	4,06
		6,00	27,63	21,69	835,21	111,36	5,50	444,72	88,94	4,01
100	200	3,00	17,41	13,67	924,37	92,44	7,29	318,26	63,65	4,28
		4,00	22,95	18,01	1199,81	119,98	7,23	410,88	82,18	4,23
		5,00	28,36	22,26	1459,51	145,95	7,17	497,19	99,44	4,19
		6,00	33,63	26,40	1703,83	170,38	7,12	577,44	115,49	4,14

The value closest to that obtained by the above calculation is that of the rectangular structural tube 50 mm x 100 mm and 4 mm thick, which has a section modulus S (W in the catalog) of 26.85 cm³ and a distributed weight of 84.3 N/m (8.59 kgf/m).

To determine the section resistance for axial loads, we determined that the maximum value of compressive force is concentrated in the BC section of the link (bar) which, as can be seen in Figure 25, occurs when the angle α of the table arms are at their minimum value of 6.6° and numerically is 21 197 N in compression.

$$\sigma = \frac{F}{A} = \frac{21\,197\text{ N}}{1\,095\text{ mm}^2} = 19,4 \frac{\text{N}}{\text{mm}^2} \text{ o } 19,4\text{MPa en compresión}$$

The safety factor for this effort is:

$$FS = \frac{35,8 \text{ MPa}}{19,4 \text{ MPa}} = 1,85$$

which indicates that it will have no problems in supporting these loads.

For the case of shear loads the bar is subjected to simple shear and the shear stress will be:

$$\tau = 1,5 \left(\frac{F}{A} \right) = 1,5 \left(\frac{1\,218 \text{ N}}{1\,095 \text{ mm}^2} \right) = 1,67 \frac{\text{N}}{\text{mm}^2} \text{ o } 1,67 \text{ MPa.}$$

The allowable shear stress for SAE J 403 1008 steel is obtained from the following relationship:

$$\tau_{\text{permissible}} = \frac{\sigma_{\text{cedencia}}}{2(8)} = \frac{286}{16} = 17,9 \text{ MPa}$$

Therefore, the safety factor is:

$$FS = \frac{17,9}{1,67} = 10,7$$

which indicates that there will be no problems due to shear stresses.

However, the combination of both axial forces, generated by pure compression and bending, and shear forces must be considered, mainly in the area where the C-joint is placed, since an orifice in that area produces a stress concentrator and a reduction in area.

6. Calculation of the pin diameter at the C-joint.

A single shear load is assumed, therefore, the minimum pin diameter will be:

$$D = \sqrt{\frac{4F}{\pi \tau_{\text{permissible}}}} \text{ ec. (0.22)}$$

If the steel selected for the pin is AISI/SAE 1045, its shear strength is

$$\tau = \frac{\sigma_{\text{cedencia}}}{2} = \frac{392,4}{2} = 196,2 \text{ MPa}$$

half the yield stress, therefore:

The allowable shear stress, for repetitive loads, shall be

$$\frac{196,2}{8} = 24,5 \text{ MPa}$$

Replacing in equation 1.22, we obtain:

$$D = \sqrt{\frac{4(21\,196,5)}{\pi(24,53)}} = 33,2 \text{ mm}$$

Therefore, the minimum diameter for the pins in joint C will be 35 mm with AISI 1045 steel (without heat treatment).

Conclusions

This work shows us the usefulness of open source scientific programming language programs to perform numerical calculations, such is the case of OCTAVE whose scripts can be compatible with those of MATLAB and can be executed in operating systems such as Microsoft Windows and GNU/LINUX. We have used well known physical principles of physics such as Newton's first and third laws for rigid bodies, which allows us to parameterize the equations that will give us the numerical values of the forces on the supports of the links (bars) that make up the mechanism of the scissors table.

Using numerical data allows us to test the performance of the equation program in the calculations and instead of obtaining a numerical table of the results we have preferred to plot these values for different positional angles of the table arms, angle alpha (α). We have also developed the programs (scripts) for the determination of the internal forces of the EF and BD arms that allow us to have the maximum values of the axial forces, shear forces and bending moments that are the basis for the determination of the dimensions of the structural elements, whether it is a rectangular tube, plates, structural angles or bars to be placed as arms of the table.

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