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Between Equations and Stories: An Encounter Between Teachers and the History of Mathematics

Entre Ecuaciones e Historias. Un Encuentro entre Docentes y la Historia de la Matemática

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Abstract

Mathematics teaching has historically faced difficulties at all educational levels, particularly in conceptual understanding rather than in the reproduction of procedures. In this context, the perceptions of high school teachers regarding the incorporation of the history of mathematics as a didactic resource were analyzed. To this end, the teachers participated in a specialized seminar in which they interacted with researchers who addressed historical, social, and cultural aspects related to the construction of mathematical concepts and theories. The results show that these perceptions can be grouped into categories that make it possible to identify relevant contributions to the integration of the history of mathematics as a pedagogical tool, as well as potential difficulties in its implementation. It is concluded that, despite certain challenges, teachers recognize that this incorporation

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helps students understand mathematics as a human activity, socially constructed over time through collective historical contributions.
Keywords: history, mathematics, human activity, teachers

Resumen
La enseñanza de las matemáticas ha enfrentado históricamente dificultades en todos los niveles educativos, especialmente en la comprensión conceptual más que en la reproducción de procedimientos. En este contexto, se analizó la percepción de profesores de bachillerato sobre la incorporación de la historia de la matemática como recurso didáctico. Para ello, los docentes participaron en un seminario especializado, en el que interactuaron con investigadores que abordaron aspectos históricos, sociales y culturales vinculados con la construcción de conceptos y teorías matemáticas. Los resultados muestran que dichas percepciones se agrupan en categorías que permiten identificar aportes relevantes para la integración de la historia de la matemática como herramienta pedagógica, así como posibles dificultades en su implementación. Se concluye que, pese a ciertos retos, los profesores reconocen que esta incorporación favorece que los estudiantes comprendan la matemática como una actividad humana, socialmente construida a lo largo del tiempo mediante aportaciones colectivas históricas.
Palabras clave: historia, matemáticas, actividad humana, docentes.

Introduction

In recent years, there has been growing interest in incorporating elements of the history of mathematics as a pedagogical resource in teaching. Numerous studies and teaching experiences have shown that including the historical context of mathematical concepts not only allows for a better understanding of their evolution but also adds a human and cultural dimension to knowledge, fostering student motivation and critical thinking. Despite this potential, the history of mathematics continues to play a secondary role in many school curricula and teaching practices.

The General Directorate of High Schools at the Autonomous University of Sinaloa (UAS) organized a seminar featuring a series of lectures titled "The Importance of the Historical Context of Mathematics in the Development of Mathematical Thinking." This initiative, aimed at mathematics teachers, sought to address various relevant historical topics, as well as to introduce key figures, pivotal moments, and iconic problems in the evolution of mathematical thinking. The lectures aimed not only to disseminate historical content but also to raise questions about current teaching practices and to encourage teachers to consider the historical development of mathematics as a resource for enriching their classes.

The main purpose of this article is to summarize the core content of this lecture series and to present the results of a qualitative study conducted with the participating teachers; their perceptions, experiences, and suggestions were gathered through questionnaires and interviews. Through the analysis of these responses, we seek to understand how this training—which differs significantly from other teacher training and professional development courses they had taken—was received; which elements were most meaningful to the participants; and to what extent they consider it relevant to integrate the historical approach to mathematics into their teaching practices. The text is organized into three parts: first, the topics covered in the lectures are presented; second, the methodology used for data collection and analysis is outlined; and finally, the main findings are discussed, followed by some conclusions and projections for future training initiatives. The goal is to promote recurring seminars on the history of mathematics based on the proposals and suggestions of the participating teachers, as well as activities involving classic problem-solving—initiatives that could foster greater interest in learning mathematics among high school students.

The history of mathematics is not merely a collection of interesting anecdotes or an understanding of the rigorous development of the mathematical concepts that comprise it; rather, with the right strategies, it can serve as a valuable pedagogical tool that could help students internalize the concepts and processes of high school mathematics.

This notion of the relevance of the historical development of mathematics in teaching has been studied by various researchers; in this regard, (Anacona, 2003) notes that "the key logical and epistemological elements in the process of theoretical construction not only enable a better understanding of the concept but also reveal characteristic aspects of the mathematical construction process, which deserve to be taken into account by teachers in their educational approaches" (Anacona, 2003, p. 30).

According to (Rengifo et al., 2015), as a result of their research project, several reasons were identified that justify the inclusion of the History of Mathematics in initial teacher education programs. Among these, a general consensus stands out regarding the relevance of this discipline in teacher preparation; the existence of instructors committed to the historical analysis of mathematics as an academic field; a favorable regulatory framework that allows for curricular modifications in bachelor's degree programs; the independent development of historical knowledge about mathematics by teachers; and the presence of study and research groups that promote theoretical and practical approaches to the historical development of mathematics and its role in teaching.

In this regard, (Dalcín et al., 2017) argue that "the historical perspective brings us closer to mathematics as a human science—not deified, sometimes painfully slow to advance, and on occasion fallible, but also capable of correcting its errors. It brings us closer to the fascinating personalities of the men who have helped advance it over many centuries." (Dalcín, 2017, p. 107). In this context, the authors argue that incorporating the history of mathematics into teaching processes can foster a more dynamic understanding of this discipline by presenting it as a social construct resulting from the collective effort of men and women over time.

According to (Ruiz, 2004), cited in (Moreira and Vera, 2023), the role of the history of mathematics in mathematical teaching and learning processes is as follows:

o A catalyst for a change in attitude toward mathematics.

o A tool for explaining and overcoming epistemological obstacles.

o An incentive for reflection and a critical attitude among students.

o A resource for integrating mathematics with other disciplines.

o An element in the training of mathematics educators.

o A means of fostering students' interest in and motivation toward mathematics.

In this discussion of the importance of including the history of mathematics in its teaching, (Dorce, 2019) argues that it contributes three very important and significant improvements: first, the history of mathematics offers a valuable framework for gaining a deeper understanding of how mathematical concepts have evolved over time.

Since the teaching of mathematics is a human activity, it is enriching to integrate it with a perspective that takes into account the social, cultural, and historical contexts in which it has developed.

4.1 Lecture 1: Why Do We Count, and Who Counts? The Social Role of Numbers from a Historical and Cultural Perspective.

Content:

In the lecture, the speaker noted that reconsidering the ideas we teach—and those from which we begin our research—means asking ourselves how they have functioned historically, not only in the generation and socialization of knowledge, but also by recognizing that these ideas belong to specific contexts. The ways in which we have constructed scientific discourse—especially from the Enlightenment to the present day—are part of a project that is also social, political, ideological, and cultural. Analyzing these constructions allows us to understand what the people who proposed certain ideas were doing, what questions they asked, what their historical context was, and why they interpreted reality in a particular way—often with an almost obsessive intensity. This perspective invites us to reflect from the present: How do we understand those scientific ideas today? Why did they emerge, and why were they important to those who developed them in their time? Finally, it leads us to fundamental questions such as:

- What does mathematics mean to people, in specific contexts, from a historical perspective?

- Why and how do we count?

Although today we view it as a mental or abstract act, the act of counting is deeply

linked to the body—particularly the fingers—and the environment. The transition from physical counting to mathematical abstraction occurred through a process of gradual separation between mathematical thought and physicality—a "detachment from the body." Counting has never been a simple act. Numbers, in their origin, are linked to concrete objects, the human body, and specific cultural needs. Our current understanding of counting has been shaped by historical narratives that, at times, oversimplify its evolution. However, both archaeology and other disciplines agree that the earliest number systems reflected very specific cultural understandings. Therefore, it is important to recognize that numbers are also cultural symbols, whose form, use, and meaning are deeply shaped by the social contexts that produce them. From this perspective, the anthropology of numbers takes on significance, as it helps us understand the impact that mathematical thought has had—and continues to have—on the organization of human societies. Let's discuss a mathematical concept whose cultural and historical dimensions are particularly relevant: the unit. From a practical standpoint, a unit represents a set of reference objects—such as a meter or a kilogram—that allows for the standardization of measurements.

This concept was fundamental to standardizing social processes such as trade, distribution, and the management of goods. For example, in ancient markets, a stone used on a scale served as a unit for weighing various products. Since that stone had a constant weight, it standardized the measurement of pistachios, almonds, chili peppers, beans, rice, and so on. People no longer sold "an almond" or "a handful of beans," but rather "a stone's weight," regardless of the specific product. This was a crucial step toward abstraction: numbers began to separate from concrete objects, and the concept shifted from the unit to the concept of "one."

In this context, the concept of unity became a symbolic measure that could be applied to multiple objects, thus marking a milestone in the development of mathematical thought.

The conference invited teachers to reflect on the historical and cultural role of mathematics, questioning the ideas we teach and the frameworks through which we conduct research. It was emphasized that mathematical concepts have emerged within specific social, political, and cultural contexts. Understanding what motivated those who developed these concepts—what questions they asked, what reality they sought to understand—allows us to redefine how we teach them today.

It was also noted that the inclusion of abstract mathematics in basic education is a recent phenomenon, with a history of barely two centuries in Europe and just over one in the Americas. This gap partly explains the current difficulties in understanding abstract concepts; as a society, we have been interacting with them in formal educational contexts for a relatively short time.

Overall, the lecture proposed viewing mathematics not only as a set of techniques but as a cultural and historical product, the study of which can profoundly enrich teaching by connecting knowledge to its human and contextual roots.

4.2 Lecture 2: From Archimedes to Newton and Leibniz. A Historical Analysis of the Creation of Calculus.

Content:

The lecture begins by highlighting the importance of mathematical thinking as a complex and multifactorial phenomenon involving historical, cultural, and epistemological elements. One of the central themes of the analysis is the epistemology of mathematics—understood as the study of the origins, evolution, and forms of mathematical concepts—which is essential for enriching teachers' training and improving their educational practice. In this regard, understanding the historical and cultural context in which concepts such as arithmetic, algebra, and calculus emerged allows for a deeper comprehension of these concepts and enables them to be conveyed in a more meaningful way.

Throughout the lecture, the contributions of major figures in the history of mathematics are analyzed, with a special focus on Archimedes, Newton, and Leibniz, while also acknowledging other key thinkers such as Euclid, Apollonius, Thales, Pythagoras, Eratosthenes, Cavalieri, Descartes, and Euler. Archimedes is presented as an exceptional mathematician whose contributions transcended his era and laid the essential foundations of modern mathematical thought. Despite his practical applications—such as the principle of buoyancy or the design of concave mirrors—Archimedes was more interested in the abstract principles of mathematics and physics.

Special emphasis is placed on Archimedes as one of the great precursors of calculus, due to his work with infinite processes and approximations, which foreshadow the modern concepts of limits, derivatives, and integrals. His discovery regarding the volume of a sphere inscribed in a cylinder is highlighted; he considered this his most important achievement—so much so that he requested that this figure be carved on his tombstone, which allowed his grave to be identified centuries later.

Regarding the Fields Medal—the highest honor in mathematics (since there is no Nobel Prize in this field)—it is noted that Archimedes appears in its design as a symbol of mathematical genius and his legacy. His image, on both the obverse and reverse, underscores his historical significance and his impact on the discipline.

As for Euclid, his vast mathematical output—more than 80 treatises—and his influence on later figures are noted. In his time, together with Archimedes and Apollonius, he helped shape what is known as one of the two great golden ages of mathematics: the first in ancient Greece; the second, in the 17th century, with Descartes and Newton as the leading figures of modern thought.

Descartes marks a key transition with the development of analytic geometry, which enabled a shift from the synthetic thinking of antiquity toward modern analytical thinking. The text highlights criticism of the excessive use of figures in traditional geometry—which exhausted the imagination—and the confusion caused by algebra in its early stages, illustrating the tensions between intuition and abstraction in the evolution of mathematical knowledge.

Euler is presented as a fundamental figure in the development of calculus, particularly for his work with functions, limits, derivatives, integrals, and infinite series. Furthermore, he devised the modern binary system—a precursor to computing—and participated in the design of calculating machines based on Pascal's studies. His method for identifying maxima and minima of functions foreshadowed what is now known as the calculus of variations.

Cavalieri, for his part, contributed the principle that bears his name, which allows for the comparison of the volumes of different solids using plane sections parallel to their bases—another key precursor to the development of integral calculus. This principle can be observed, for example, in the calculation of areas using lower and upper rectangles, as taught in high school under the topic of cumulative changes.

Isaac Newton is discussed not only as a mathematician but also as a physicist, astronomer, alchemist, and scholar of the Bible. His contributions to differential and integral calculus—along with Leibniz, with whom he had a famous controversy over the authorship of calculus—are fundamental. In addition, he developed Newton's binomial theorem, and his work on dynamics and universal gravitation marked a watershed moment in the history of physics. It is noted that, ironically, although Newton disliked the term "derivative," the slope of a tangent to a curve—an essential concept in calculus—is precisely a derivative.

In the final section, the speaker points out that calculus is the mathematical theory used to solve problems involving phenomena in motion, whether physical (such as the motion of the moon, the sun, or the Earth) or social and economic (such as market behavior or the value of the dollar). Calculus allows us to formalize and predict these changes, demonstrating its applicability in fields as diverse as biology, chemistry, nuclear physics, economics, and many other disciplines.

Taken as a whole, the lecture presents a historical, epistemological, and pedagogical overview of the evolution of calculus, highlighting how the contributions of great thinkers from antiquity to modern times not only transformed mathematics but also gave rise to new ways of understanding the ever-changing world.

4.3 Lecture 3: Pythagoras, Hippasus of Metapontum, and the Discovery of Irrational Numbers. The crisis triggered by .

Content:

The lecture addressed one of the most dramatic and intellectually challenging moments in the early history of mathematics: the discovery that the square root of 2 cannot be expressed as a fraction—that is, that is an irrational number. This discovery, traditionally attributed to Hypatus of Metapontum, a member of the Pythagorean school, sparked a genuine philosophical-mathematical crisis within that community, as it called into question one of its fundamental pillars: the belief that everything in the universe could be explained through whole and rational numerical relationships.

The root of the conflict lay in the fact that, although geometry allowed for the construction of line segments such as one of length √2 (the diagonal of a square with side length 1, for example), it was not possible to represent that measure as the quotient of two integers. This was especially problematic for the Pythagoreans, whose conception of number was not yet completely separated from the concept of concrete measurement.

The discovery that √2 is irrational required a huge conceptual leap: accepting that not everything measurable in geometry was reducible to fractions or rational numbers, which implied that the universe, from their mystical and mathematical perspective, could not be fully organized through arithmetic.

The presentation explored how this discovery was met with hostility within the Pythagorean community, to the extent that, according to tradition—though not fully historically verified—Hypasos was punished, exiled, and even killed for revealing this "disruptive" knowledge to the outside world. Beyond the historical accuracy of these events, what matters is the symbolism: the clash between dogmatic belief and mathematical reality. Aspects of the Pythagoreans' daily life were also discussed: their closed-off structure, rituals, hierarchies, and their belief in the transmigration of souls and the mathematical harmony of the cosmos. A particularly interesting part of the lecture centered on the contrast between the abstract world of numbers and the visual and tangible experience of segments in geometry. Thus, although a segment of meters looks just as "normal" as one of 1.5 meters, the difficulty lies in the fact that the former cannot be expressed as an exact fraction. This difference was difficult to grasp in antiquity, since geometry was visual and concrete, whereas irrational arithmetic was an abstraction with no exact representation at that time.

Finally, the lecture reflected on the legacy of the discovery: although it was initially an intellectual scandal, irrationality opened the door to a richer and more complex conception of numbers and foreshadowed later developments such as real numbers, transcendental irrational numbers, and mathematical analysis.

The speaker emphasizes that, unlike many other philosophical and scientific schools of antiquity, the Pythagorean school did allow—and even valued—the participation of women. Pythagoras believed that both men and women could attain virtue through study, discipline, and the philosophical life, which was very progressive for his time (6th century B.C.). It is known that at least 17 women were part of the Pythagorean school, and some ancient sources mention that they even wrote treatises (although none have survived). Theano was likely Pythagoras's wife, although some sources present her as his disciple. She is credited with writings on ethics, mathematics, physics, and medicine. Although no original texts of hers have survived, tradition attributes to her treatises such as On Virtue and On the Golden Ratio. She is particularly associated with the idea of the "golden mean" (or golden ratio), although it is unclear whether she was actually the one who formulated it, since much of what is attributed to her may have been transmitted later by other Pythagoreans. She has served as a symbol of the intellectual role of women in antiquity and has inspired both historians and feminist philosophers.

4.4 Lecture 4: The Encounter Between Algebra and Geometry in Antiquity and Modernity: Euclid and Descartes.

Content:

The lecture addressed Euclid's fundamental role in the systematization of geometric knowledge in antiquity. Through his work The Elements, Euclid succeeded in establishing a logical and rigorous model based on the axiomatic-deductive method, in which a small set of definitions, postulates, and common notions led to the deduction of hundreds of geometric propositions. This work not only influenced the mathematical thinking of its time but also became one of the most widely reproduced and studied scientific texts of all time, serving for more than 2,000 years as the foundation for the teaching of geometry.

It was emphasized that Euclid's influence was so profound that his way of conceiving and organizing geometric knowledge became the quintessential model of science, inspiring even thinkers such as Newton, Spinoza, and Hilbert. In antiquity, The Elements represented a form of structured rational thought that contrasted with mythological or empirical explanations, and was considered an expression of the order of the cosmos reflected in human thought. For this reason, Euclidean geometry was not merely technical knowledge but a form of education for the intellect and the spirit, highly valued in the philosophical academies of the ancient world.

In contrast, the modern era brought with it a new way of conceiving geometric space, particularly through the work of René Descartes, who in the 17th century proposed the unification of algebra and geometry through the Cartesian plane. With this contribution, analytic geometry was born, allowing geometric figures to be described using algebraic equations and making it possible to study curves and objects that could not be addressed with purely Euclidean tools.

The lecture highlighted the key differences between Euclidean and analytic geometry: This convergence of algebra and geometry was crucial for the development of calculus, mathematical physics, and various branches of modern mathematics. The introduction of the Cartesian plane not only provided technical advantages but also changed the way space and motion are conceived, paving the way for the study of more complex geometric objects and greater abstractions, such as algebraic curves, n-dimensional spaces, and differential geometry.

While the former focuses on geometric constructions and visual or spatial reasoning,

the latter translates geometric problems into a symbolic algebraic language, facilitating calculation, generalization, and graphical representation.

An emotional closing:

At the end of the conference, the speaker shared a link with attendees to a PDF file containing Euclid's book The Elements. To the surprise of many, it was the first time that several teachers (some with more than 20 years of experience teaching Euclidean geometry) had direct access to this work. This gesture sparked emotion and reflection among those present, highlighting that, in the day-to-day teaching of mathematics, the historical dimension of knowledge is often overlooked, despite its enormous potential to enrich students' understanding and appreciation of the discipline.

4.5 Lecture 5: The Use of the History of Mathematics in Mathematics Education.

Content:

The speaker began by noting that he has been fully convinced since his own student days that mathematics should be taught through its historical evolution.

This lecture analyzed how the history of mathematics can be a powerful tool for improving teaching and learning processes. Understanding the historical development of mathematical concepts allows students to better grasp their origins, their usefulness, and their construction as part of an evolving culture. This perspective humanizes mathematics and makes it more accessible.

Reading skills and reasoning are fundamental to learning mathematics. In this regard, knowledge of the history of mathematics can be used by teachers as a tool to motivate students to read articles, books, or chapters appropriate for their age level. There are numerous engaging and accessible options for different grade levels.

He made a very interesting point: we must dispel the notion among teachers and students that the humanities are very simple compared to the natural and exact sciences. This is because history is not merely about recounting or describing past events, but rather about understanding the context of those events and—as in the case of the history of mathematics—how we can use contextual knowledge of the development of the mathematical concepts we teach as a teaching tool.

One of the key points was that Euclid's method of proof, as outlined in the book *The Elements*, while fundamental to the history of mathematics, is complex to use in the classroom due to its logical and formal demands, which presents a pedagogical challenge. However, this axiomatic-deductive approach has great advantages, as it shows us how many more ideas can be derived from just a few and deduced in logical order. Logical order in mathematics is essential; it teaches us how to think. The speaker also noted that the high school math curriculum covers many different topics, and it is natural for students not to feel equally comfortable with all of them: they may show an affinity for some and a dislike for others. Furthermore, it was emphasized that anecdotes about mathematicians and their lives can make class more interesting, especially when the teacher discusses them in context. For example, it was recounted that when Euclid heard a student ask, "What will I get out of learning geometry?", he gave the student a coin and told him that, since he was looking for a benefit, he could have this coin as a reward. These kinds of stories spark curiosity, bring students closer to historical figures, and reveal the human side of mathematics.

In conclusion, it is noted that incorporating the history of mathematics does not mean adding difficult content, but rather enriching what is already being taught. Showing the mistakes, debates, and explorations behind each mathematical idea helps students understand that mathematics was not born fully formed, but was built up little by little. This not only facilitates conceptual understanding but can also increase motivation and interest in learning. It is noted that Euclid's *Elements* is the most important textbook in history, with numerous translations; however, a complete Spanish translation was not available until the 20th century.

4.6 Lecture 6: Looking Back: Numbers and Their Arithmetics.

Content:

The lecture began with a critical reflection by the American historian Jacques Barzun, who observed that algebra often becomes off-putting to students because teachers do not explain the "why" behind what they teach. Mathematics is frequently presented as a finished, abstract system, devoid of context or history, which gives the impression that it was "dropped from the sky" to be used by a select few geniuses.

In contrast, the speaker emphasized that the use of the history of mathematics, when properly linked to current knowledge, is a powerful teaching tool. This approach allows for teaching the whys behind mathematical concepts, facilitating a deeper understanding.

The classification by Jones (1969) regarding possible approaches to integrating history into mathematics education was revisited: It explains the historical development of mathematical concepts and answers questions such as: These questions can be used to discuss the arbitrariness of definitions, the evolution of the concept of number, and the cultural and psychological foundations of mathematical systems. It focuses on tracing the origin of mathematical ideas and their logical evolution. For example: It involves drawing on historical elements to select teaching strategies. It allows us to understand how concepts were taught in the past and how they have evolved. Old textbooks contain instructional sequences and definitions that can inspire current practices.

Chronological use:

Why does a day have 24 hours?

Why are there 60 minutes in an hour?

Logical (or genetic) use:

Descartes avoided negative numbers and called them "false."

Gauss was suspicious of infinity.

Newton wrote several versions of his "fluxions" because he was dissatisfied with their theoretical foundation.

Hamilton pioneered the use of ordered pairs to define complex numbers.

This approach contributes to the development of logical reasoning and the understanding of axiomatic systems.

Pedagogical use:

Origins of the concept of number

Archaeological findings were presented that show how humans have perceived and represented quantities since prehistoric times:

o The Lebombo Bone (35,000 years old): It contains 29 marks possibly related to the lunar or menstrual cycle.

o The Ishango Bone (over 20,000 years old): found near the source of the Nile, it shows groups of notches representing quantities.

These artifacts demonstrate a transition between two levels of abstraction: counting and recording what has been counted. The ability to represent quantities distinguishes humans from other species.

Processes that led to the development of the concept of number:

Freudenthal proposes three stages in the evolution of the concept of number: They appeared about 5,000 years ago. They are sets of symbols (numeral signs) used to record and perform operations with numbers. They are classified as: Multiple ways of understanding number

Enumeration:

This involves counting objects one by one, without the need for number words.

Numeration:

This arises when body parts are replaced by words. Spoken language allows quantities to be represented symbolically and sequentially.

Number systems:

Additive

Hybrid

Positional

Freudenthal emphasizes that the expression "the concept of number" is misleading, since there are different types of numbers, with varying meanings depending on the context and the pedagogical perspective: It arises from the act of enumerating objects. It is formalized in the natural numbers and in Peano's axioms. At a more advanced level, it gives rise to transfinite ordinal numbers (Cantor). It allows us to identify the number of elements in a set (cardinality). It is even observed in animals. Its mathematical culmination is cardinal infinities. It is associated with the comparison of magnitudes. It involves fractions and decimals, and is formalized in rational numbers and, subsequently, real numbers. It can be extended to non-Archimedean fields. It refers to numbers as objects that can be manipulated through operations. It is formalized in rings and fields in algebraic theory.

Counting number:

Number of numerosity:

Measuring number:

Calculating number (algorithmic):

Educational implications:

It was emphasized that older textbooks are a source of useful strategies, as they contain:

o Proven instructional sequences,

o Specific treatments of content, and

o Historical definitions that show how concepts have evolved.

Freudenthal also noted that students often fail to grasp large numbers (such as "billions") and that meaningful mathematics education must focus on understanding what numbers mean in real-world contexts.

In conclusion, the conference made it clear that incorporating history into mathematics education:

o Enriches conceptual understanding,

o Humanizes the content,

o Stimulates student curiosity,

o And provides teachers with solid pedagogical tools.

Numbers, far from being a single, static concept, are complex human constructs that have evolved alongside culture and deserve to be taught as such.

Methodology

This study takes a qualitative, descriptive, and interpretive approach, as it focuses on teachers' understanding of the content covered in a series of six lectures on the history of mathematics offered as part of the seminar "The Importance of the Historical Context of Mathematics in the Development of Mathematical Thinking," and to explore the perceptions, evaluations, and impacts these lectures had on the participating teachers.

The purpose is to describe and interpret the participants' experiences and reflections regarding the use of historical context in mathematics instruction, as well as to investigate which topics they consider most important in the history of mathematics and why, with the aim of continuing this type of professional development. The sample consists of teachers who instruct various mathematics courses and who voluntarily attended the lecture series. Participants were invited to complete the research instruments once the series had concluded.

An open-ended questionnaire was used, designed to gather opinions, prior experiences, and suggestions related to the use of the history of mathematics in educational contexts. The questionnaire contains the following questions:

1. What is your opinion on the inclusion of historical context in mathematics education?
2. Do you consider it relevant to incorporate the history of mathematics into your classes? Why?
3. In your experience, what benefits have you observed from using historical elements in mathematics instruction?
4. In which subjects or topics have you integrated elements of the history of mathematics? Please provide specific examples.
5. During the lecture series, which talk caught your attention the most, and why?
6. Is there a mathematician whose history or contributions have influenced your teaching style? Explain how.
7. What topics would you like to see addressed in future seminars on the history of mathematics, and why do you think they would be useful for teachers?

These questions allowed us to explore both the general value placed on historical context in teaching and specific experiences of integrating historical elements into the classroom.

After the lecture series concluded, the questionnaire was distributed to participants, who completed it in writing and anonymously. Subsequently, the responses were collected and analyzed qualitatively.

A thematic content analysis was conducted using open coding and categorization of the responses. This made it possible to identify patterns in teachers' perceptions, such as their assessment of the historical context as a pedagogical resource, the most influential figures, the subjects in which history has been integrated, and their expectations regarding future seminars.

Results

In this section, we present the results of the instrument's implementation with UAS high school teachers. Based on the information obtained from the teachers, we created the following categories in accordance with the teachers' diverse perceptions regarding the history of mathematics as a tool to support students' understanding of concepts.

The research instruments were administered to 49 UAS high school teachers.

1. On the Historical Context of Mathematics

Based on the analysis of the responses, we were able to identify various opinions regarding the historical context of mathematics, which can be grouped into four main categories:

- Generally positive assessment (they find it interesting or enriching).

- Indifference or lack of awareness (had not considered it before).

- Negative perception (consider it unnecessary or distracting).

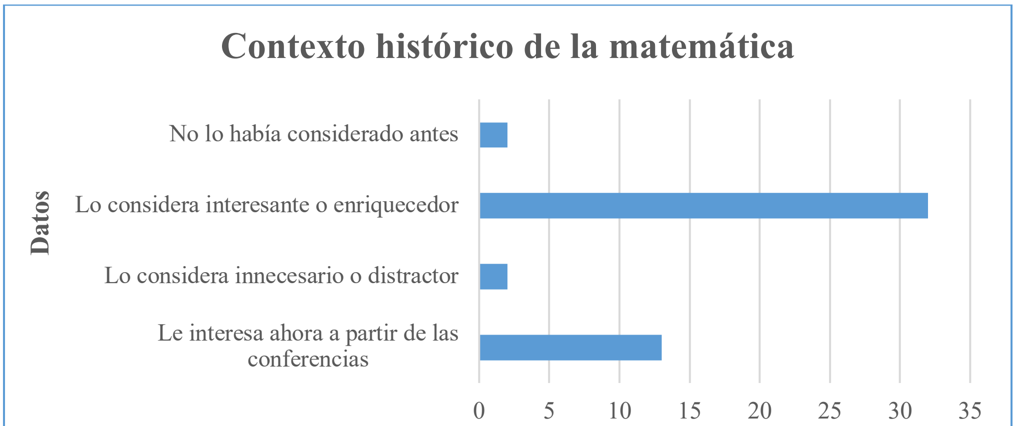
- Piqued curiosity (now interested as a result of the lectures).

The first category is a general positive assessment, in which participants express that they consider this approach interesting, enriching, or even fundamental to teaching. In contrast, some expressed indifference or lack of awareness, as they had never considered it before or fail to see its usefulness in the classroom. A negative perception was also identified among those who believe that including historical aspects is unnecessary or may be distracting from current mathematical content.

Finally, an intermediate category emerged, termed "awakened curiosity," in which attendees showed a newfound interest in the topic following the lectures, recognizing its educational potential even though they had not previously explored it.

Figure 1 shows the frequencies of questions related to opinions on the historical context of mathematics.

Figure 1. Historical Context of Mathematics



	Le interesa ahora a partir de las conferencias	Lo considera innecesario o distractor	Lo considera interesante o enriquecedor	No lo había considerado antes
Total	13	2	32	2

Source: Author's own work

This information was obtained from the teachers through questionnaires and interviews conducted after the lecture series.

The results show that, in the teachers' overall perception of the history of mathematics as a teaching aid, the "positive evaluation" category was the most frequent (32 teachers). Those in the "indifference" or "lack of awareness" category (who had not considered it relevant to include the historical context in their classes) can be considered potentially interested because, although they actively participated in the seminar, they did not demonstrate a clear willingness to use this resource, nor did they perceive it as an unnecessary distraction.

A particularly relevant finding is that 13 teachers reported not having included the historical context of mathematics in their teaching strategies but expressed interest in incorporating it after participating in the lecture series. In contrast, among those with a negative perception—who view the history of mathematics as unnecessary or a distraction—only two opinions were recorded, whose authors stated in the interview:

- Teacher 1: The curriculum is very extensive; I don't think it's important to spend time ensuring students know who invented this or that—that time is better spent on practice.

- Teacher 2: Students aren't interested in reading mathematical theory; they prefer to solve problems or have the procedures they've already covered explained to them again.

1.1 The Relevance of Incorporating the History of Mathematics into Classes and Why

Categories:

- Improving conceptual understanding

- Making the subject more appealing and motivating students

- Humanizing mathematics (showing that it is the product of cultural evolution)

- Educational and critical value (stimulating reflective, historical, or ethical thinking)

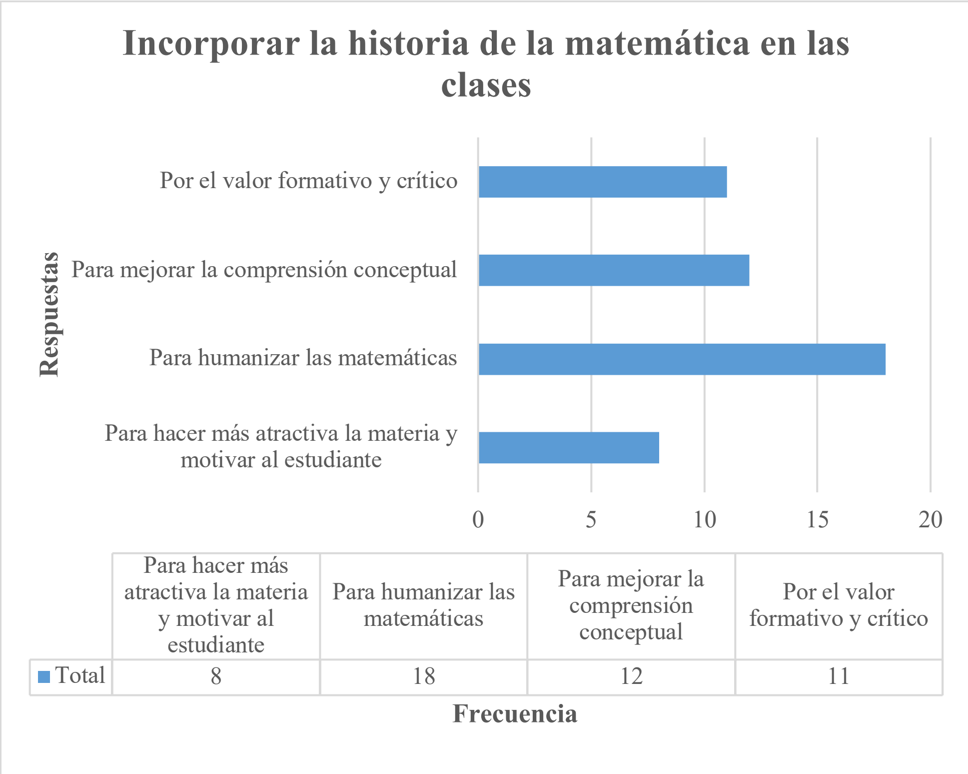
Responses regarding the relevance of including the history of mathematics fall into four pedagogical and educational categories. One of these highlights the improvement of conceptual understanding by helping students grasp the origin and meaning of mathematical concepts.

Another common reason is to make the subject more appealing and motivate students, as historical narratives generate greater interest and a stronger connection to the content. The goal of humanizing mathematics also stands out, showing that it is not an isolated construct but rather the result of a cultural evolution driven by real people who faced concrete problems.

Finally, several teachers emphasize the formative and critical value of history, as it stimulates reflective, historical, or ethical thinking in students, thereby enriching their well-rounded education.

Figure 2 shows the frequency of questions related to opinions about incorporating the historical context of mathematics into courses.

Figure 2. Incorporation of the History of Mathematics into Classes



Source: author's own work

Based on the guidelines of the New Mexican School (NEM), at the high school level, special emphasis has been placed on highlighting the human dimension of science during the various training sessions provided to teachers, so that they may understand, internalize, and correctly apply the NEM's new educational model.

In this context, these provisions may influence the perception of the history of mathematics as a teaching resource that fosters students' scientific knowledge, given that the most frequently cited category is "humanizing mathematics," followed by "conceptual understanding"—which is consistent, as teachers seek tools to facilitate the teaching and learning of traditionally complex mathematical concepts

In the "formative and critical value" category, some of the teachers' comments stand out, such as:

- The history of mathematics helps students value it as part of their holistic development; in addition to being a theory and a tool, it is part of general culture.

- Through history, students understand that mathematicians devoted a great deal of time and effort to creating or inventing concepts in algebra, geometry, and so on.

- Reading about the history of mathematics helps students better understand their professional calling.

The category of "making the subject more appealing" was mentioned less frequently than the others, contrary to what we expected. Generally, teachers perceive the history of mathematics as an opportunity to share interesting anecdotes about mathematicians and cities that were the setting for mathematical discoveries, among other things.

2. Benefits Observed by Teachers When Using Historical Elements in Mathematics Instruction

Categories:

- Helps capture students' attention

- Encourages students to read about mathematical topics

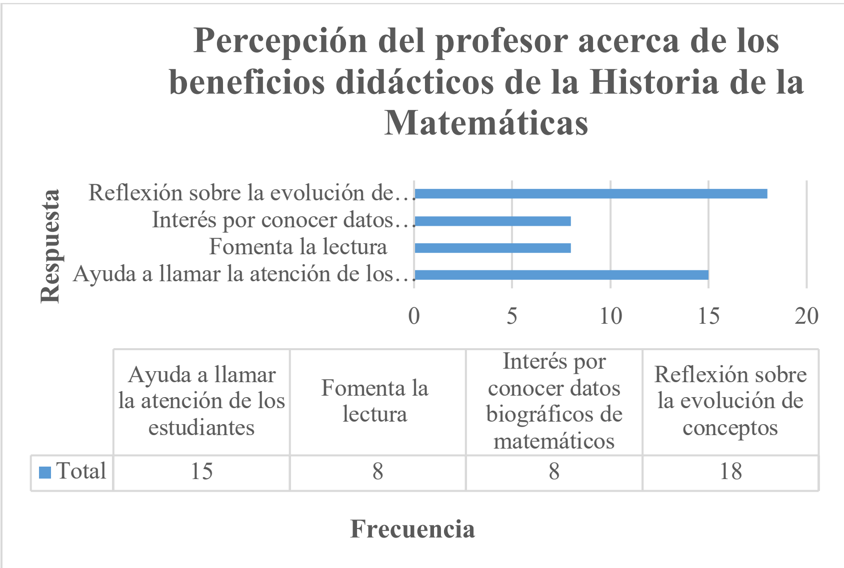
- Develops students' interest in learning about the biographical details of mathematicians

- Students reflect on the evolution of the mathematical concept being studied, noting conceptual differences

Regarding the benefits of incorporating historical elements into mathematics instruction, teachers' responses fall into categories that reveal different impacts on students, whether based on prior experiences or expectations regarding future classroom application.

The most common category is "capturing students' attention," as historical anecdotes, contexts, and stories spark an initial interest that encourages participation. Another benefit noted is the promotion of reading about mathematical topics, which fosters a broader approach to knowledge, going beyond technical exercises. Some teachers have observed that this strategy sparks students' interest in the biographical details of mathematicians, linking the subject to stories that humanize the content. Furthermore, it allows students to reflect on the evolution of mathematical concepts, analyzing how the same object—for example, the concept of number—could be conceived differently in ancient times than it is today. Figure 3 shows the frequency of questions related to opinions about incorporating the historical context of mathematics into courses.

Figure 3. Teacher Perceptions



Source: author's own work

It is interesting to note that several teachers consider it important that the history of mathematics allows students to reflect on the evolution of concepts; below are some opinions on this matter:

- In a course on the history of mathematics, we studied the string table as it was used in antiquity—which marked the beginning of trigonometry. It took me a lot of effort to understand trigonometric ratios that way; it's very interesting to learn how what we teach today originated.

- If we show the historical evolution of a mathematical object—for example, an equation or the area under the curve—we can appreciate the advantages we have today in contrast to how people worked in the past. Today we simply have pen and paper; in the past, they worked with sand. Now we can demonstrate countless examples in dynamic geometry. The passage of time allows us to reinterpret and better understand these concepts.

- It is very difficult to provide many examples of how a mathematical concept evolves, but if we explain at least one well, students can appreciate the changes and see that mathematics was built little by little through the work of many people—that it has a historical development.

From a research perspective, this reflection by teachers is relevant because, although mathematical objects (numbers, functions, triangles, etc.) remain abstract entities, their conception, representation, and teaching have changed throughout history, as have the ideas and tools that mathematicians have developed to understand them.

In this regard, we believe that if the history of mathematics is systematically incorporated into high school courses, students can appreciate the meaning we ascribe today to a mathematical object—a meaning that is the result of historical processes, discussions, and reinterpretations, as noted by Mejias (2023) and Hernández (2010).

Regarding the other three categories, teachers generally indicated a connection between encouraging reading and using biographical details and anecdotes, which together help generate interesting starting points for a class, thought-provoking questions, and so on. This implies that teachers can use the historical development of mathematics to capture students' attention—which was the second most frequently cited category—as part of the class opening or introduction.

3. Examples of Teachers Integrating Elements of the History of Mathematics into the Classroom

Categories:

- The teacher has not used historical information in their classes

- Has incorporated historical moments in mathematics

- Reviews the history of calculus (Newton and Leibniz)

- Geometry (biographies of mathematicians)

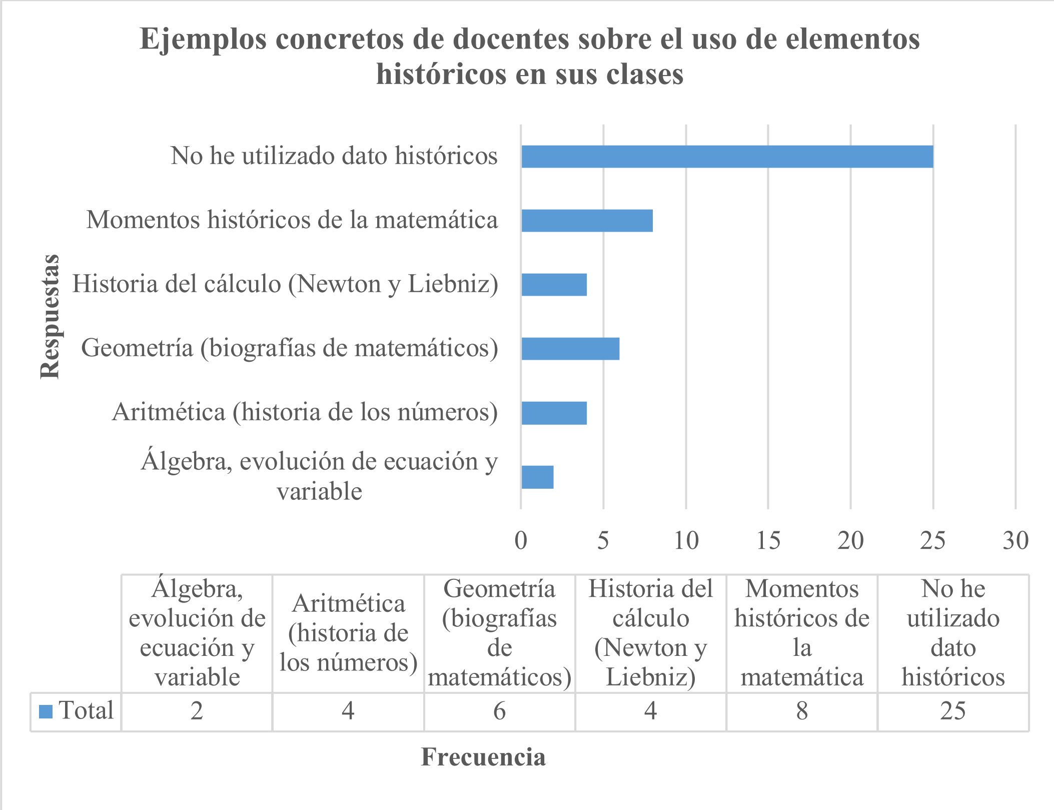
- Arithmetic (history of numbers and number systems)

- Algebra (evolution of the concepts of variables and equations)

Figure 4 shows the frequencies for this category and how many teachers fall into each subcategory.

The most frequent category is that in which the teacher does not incorporate historical information into their classes, which is to be expected given that, at this institution, using the historical context of mathematics as a teaching resource is not part of the teaching culture.

Figure 4. Examples of historical elements used by teachers



	Álgebra, evolución de ecuación y variable	Aritmética (historia de los números)	Geometría (biografías de matemáticos)	Historia del cálculo (Newton y Liebniz)	Momentos históricos de la matemática	No he utilizado dato históricos
Total	2	4	6	4	8	25

Source: author's own work

The most frequent category is that in which teachers do not incorporate historical information into their classes, which is to be expected, given that at the institution it is not part of the teaching culture to use the historical context of mathematics as a teaching resource.

However, taking into account the opinions of teachers who have incorporated historical elements into their classes, we find various categories in which it is common for teachers to use key historical moments in mathematics, as well as biographical information about renowned mathematicians. Some comments on this:

- In geometry and trigonometry, I like to talk to students about Pythagoras and also about the Cartesian plane.

- In probability, we examine the historical development leading to the law of large numbers, as well as the biographies of Gauss and Laplace.

- I have incorporated historical contexts primarily in Mathematical Thinking I and II. For example, when discussing the Pythagorean Theorem, I take the opportunity to explain its origins in the Pythagorean school; when covering real numbers, I also discuss Thales of Miletus.

- The figure I most enjoy discussing in my classes is undoubtedly Newton, due to his contributions to both mathematics and physics; he is the figure I focus on most, since I am also part of the physics department.

- Although there isn't a specific high school course dedicated to the history of mathematics, the truth is that it's indirectly included in every subject, since each course teaches the techniques and methods that have been used over the years—from their origins to how they've evolved. For this reason, in all the courses I've taught, I mention great mathematicians such as Pythagoras, Euclid, Leibniz, and Newton, to name a few.

- When teaching the Cartesian plane, I discuss with students how Descartes sought a way to represent algebraic equations geometrically, and how the idea arose by relating numbers to positions on a plane, using two perpendicular lines as a reference.

- I review the biographies of Pythagoras, Euclid, and Descartes, as they were great thinkers in the field of geometry.

4. Preferred Topics for Seminar Lectures

Lectures:

1. Why Do We Count, and Who Counts? The Social Role of Numbers from a Historical and Cultural Perspective

2. The Convergence of Algebra and Geometry in Antiquity and Modern Times (Euclid and Descartes)

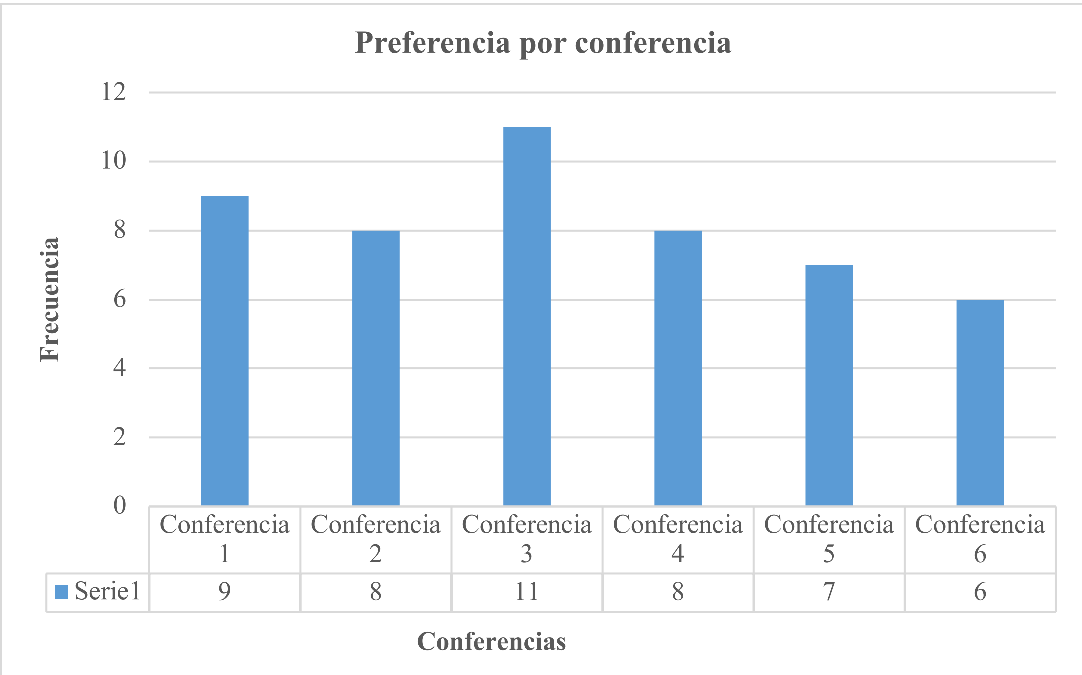
3. The Use of the History of Mathematics in Mathematics Education

4. Pythagoras, Hypatus of Metapontum, and the discovery of irrational numbers. The crisis caused by the square root of 2.

5. From Archimedes to Newton and Leibniz. A historical analysis of the creation of calculus.

6. Looking back: numbers and their arithmetic

Figure 5. Lecture preferences



Source: author's own work

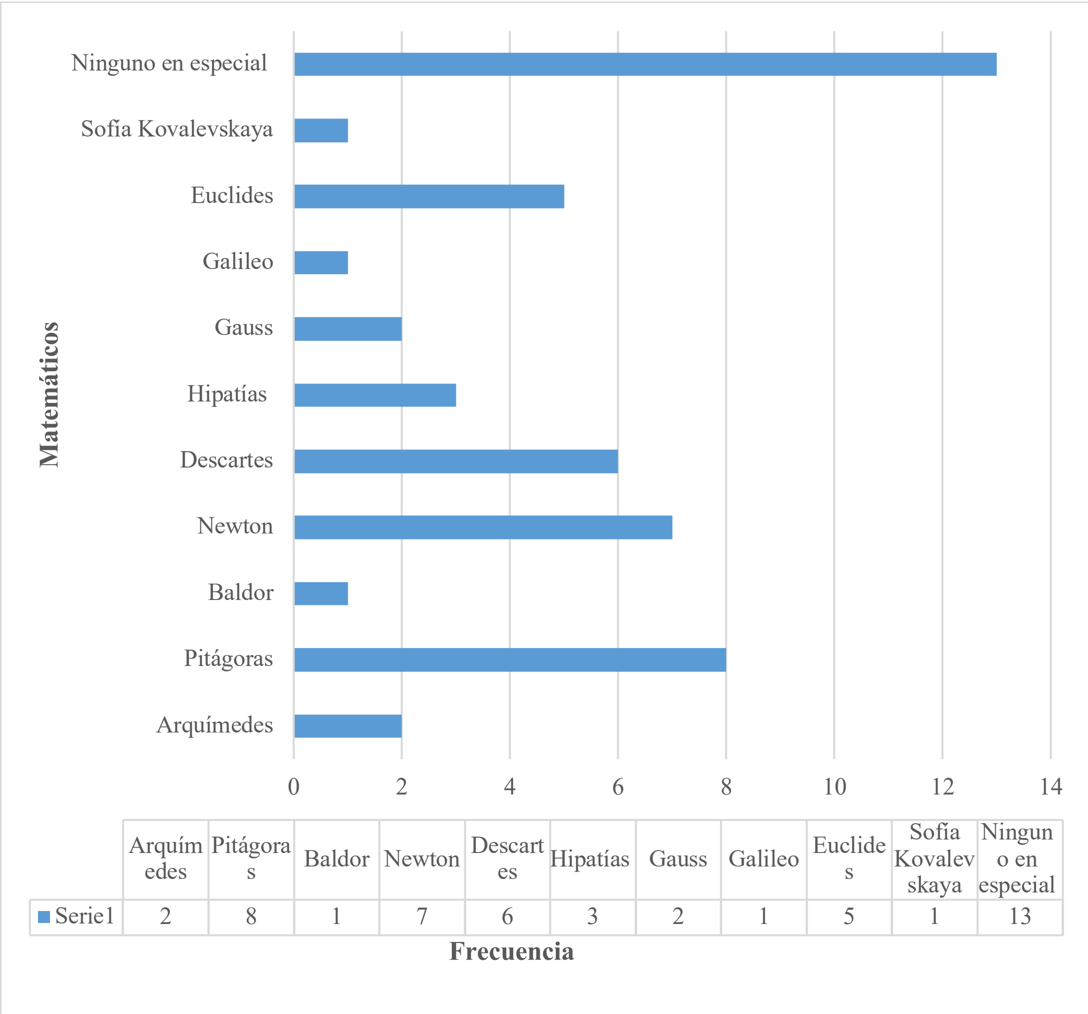
The lecture The Use of the History of Mathematics in Teaching was rated highest by participants, highlighting the importance teachers place on integrating the historical dimension into the teaching of the discipline. This preference reflects that, from the start of the seminar, there was an expectation to gain knowledge about the historical development of mathematics in order to incorporate it as a teaching resource. Although all the presentations addressed this objective, attendees noted that the third lecture offered distinctive value, particularly due to its detailed analysis of the evolution of mathematics textbooks, from Euclid's The Elements to contemporary editions.

This approach allowed teachers to reflect on how changes in the presentation of content respond to specific cultural, pedagogical, and epistemological contexts, and how such changes can inspire innovative strategies for teaching today.

5. Mathematicians Who Have Influenced Teachers to Improve Their Teaching

Participating teachers identified the mathematician whose contributions or biographical details have influenced their teaching practice; the following graph shows their preferences in this regard. It highlights that a significant number of teachers do not consider any particular mathematician to be relevant.

Figure 6. Mathematicians considered relevant from the perspective of teaching pedagogy



Source: author's own work

Notable comments on this topic include:

- Archimedes, because he laid the foundations of calculus in antiquity.

- Pythagoras, without a doubt, since the Pythagorean theorem is, by its very nature, the symbol of mathematical abstraction, and through this theorem we can apply and teach geometry in a simple way using this algorithm.

- Hypatia; I tell my students about the importance of women's role in mathematics during an era when women had neither a voice nor a vote.

- René Descartes, for algebraizing geometry. I also admire Leibniz for his passion for analyzing infinitesimal variations, starting from the secant line to the tangent line using limits.

- Archimedes and Euclid strike me as very interesting mathematicians; Archimedes because he solved the problem of the king's crown in a very clever way.

- Yes, it's inspiring that women are among the great contributors to mathematics, especially since it has traditionally been a male-dominated field. Stories and contributions like those of Sofia Kovalevskaya are inspiring for young female students, showing that it's possible for everyone to find a place in the world of numbers.

- Gauss—his story has served as one of many examples for me when teaching classes

- Pythagoras: spirituality linked to mathematics; he taught that the study of numbers elevates the soul

- Known as the father of geometry, EUCLID has significantly influenced my teaching of mathematics

- Galileo. He suggested that the universe can be expressed in mathematical terms

6. What topics would you like to see addressed in future seminars on the history of mathematics, and why do you think they would be useful for teachers?

We classified the responses regarding the topics teachers would like to see addressed in future lecture series on the history of mathematics into the following categories, and present the preferences indicated by the teachers.

Women in Mathematics

- Lectures about women who worked in mathematics

- Women in the history of mathematics

- I saw the movie *Agora*; I'd like to learn more about Hypatia

- The role of women in the history of mathematics, such as Hypatia and Sophie Germain—this promotes gender equity in scientific contexts and serves as an inspiration.

- Mexican or Latinx women mathematicians

- Women in the history of mathematics

- A lecture on Hypatia; her legacy is very interesting

Biographies

Pythagoras, Descartes, Newton, Archimedes, Gauss

Disciplines

- Historical facts about geometry and algebra

- The evolution of number systems

- History of analytic geometry

- Ptolemy's Chord Tables

- The Rivalry Between Newton and Leibniz

Miscellaneous

- The Impact of Mathematics on Technological Advances

- Mayan Mathematics: The Invention of Zero

- I'd like to know the origin of pi, its history, and why it has that symbol

- How Zero and Negative Numbers Came About

- The Pythagoreans

- The golden ratio

- I don't have any specific topic in mind; I find any topic interesting

- Any topic in the history of mathematics can be useful to us

The results show that the participating teachers valued the history of mathematics as a useful resource for enriching their teaching practice, especially because it allowed them to understand concepts from an evolutionary and human perspective. This is consistent with the findings of (Tzanakis and Arcavi, 2000), who argue that the history of mathematics can strengthen teacher education by offering a more critical and reflective view of mathematical objects. Similarly, Radford (2011) has emphasized that incorporating historical aspects helps teachers recognize the importance of cultural context in the construction of mathematical knowledge.

As in (Azman and Maat, 2021), the findings show that mathematics teachers hold a positive view of incorporating the history of mathematics into teaching and learning processes. However, its implementation in the classroom is limited by various factors, including a lack of specific knowledge and skills needed for its integration, a shortage of materials, and the absence of history in the curriculum and in assessments for mathematics courses.

Lack of time is another difficulty cited by teachers when it comes to incorporating the history of mathematics into their classes. They believe that the time available is barely sufficient to cover the content of the courses they teach, a situation that aligns with the findings of (Jankvist, 2009), who identifies lack of time as one of the main obstacles to integrating history into mathematics courses. This suggests that the history of mathematics should be viewed as a teaching resource capable of strengthening conceptual mastery of the mathematical objects under study, as it is a strategy that promotes understanding of concepts and, consequently, improves the use of algorithms and procedures, which directly impacts problem-solving.

Conclusions

Teachers' participation in the seminar on the history of mathematics is significant for many reasons. In particular, it allows for an understanding of mathematical objects from a human perspective, by recognizing that they did not emerge as fully formed entities from the outset, but rather were constructed over the course of a conceptual evolution. This process was marked by debates among mathematicians, informal proofs, refutations, reconstructions, and reconsiderations of previously accepted claims. However, although teachers view the incorporation of the history of mathematics into their courses as definitively positive, the question then arises: How can the history of mathematics be effectively incorporated into courses so that it is not limited to being an anecdotal supplement, but rather becomes a teaching resource capable of directly influencing a deep understanding of mathematical concepts?

At the same time, its integration should help students view mathematics not merely as an abstract body of knowledge, but as a constantly evolving human construct, shaped by specific historical, social, and cultural contexts. In this way, the time spent incorporating it into mathematics courses will be reflected in curricular progress and will not be a waste of time. To this end, we believe research is necessary to identify such teaching strategies, in accordance with the high school curriculum in Mexico and the guidelines of the New Mexican School, which repeatedly emphasize the need to showcase the human side of the sciences.

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