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Comparative research study based on the use of the two-color geometric algebra technique to develop spatial thinking

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Abstract

The comparative analysis carried out in this work allows us to determine the differences and similarities of two investigations, in which didactic proposals are presented with the teaching strategy: Geometric Algebra in Two Colors. This analysis, together with the results of the interviews with the teachers of Mathematics I, and the results obtained in the evaluation of the first semester students of Business Administration and Accounting and Auditing, allow us to prove that the application of this didactic proposal helps to develop spatial thinking and to prove the hypothesis, which assumes that two-color geometric algebra helps to improve academic performance in mathematics.

Keyword: methodology, didactic proposal, mathematics education

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Técnica álgebra geométrica a dos colores para desarrollar el pensamiento espacial

Resumen

El análisis comparativo realizado en este trabajo nos permite determinar las diferencias y similitudes de dos investigaciones, en las que se presentan propuestas didácticas con la estrategia de enseñanza: Álgebra Geométrica a Dos Colores. Este análisis junto a los resultados de las entrevistas dirigidas a los docentes que imparten la asignatura de Matemáticas I, y a los resultados obtenidos en la evaluación tomada a los estudiantes de primer semestre de las carreras de Administración de Empresas y Contabilidad y Auditoría, nos permiten comprobar que la aplicación de esta propuesta didáctica ayuda a desarrollar el pensamiento espacial y a comprobar la hipótesis planteada, la misma que tiene como supuesto que el álgebra geométrica a dos colores ayuda a mejorar el rendimiento académicos en las matemáticas.

Palabra clave: metodología, propuesta didáctica, enseñanza matemática

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INTRODUCTION

One of the main obstacles in the learning of mathematics is the poor mastery of basic concepts, which leads to insecure and ineffective processes for solving exercises and makes it difficult to anchor with the topics foreseen in higher levels of the curricula.

The effective learning of students and the formation of professionals capable of solving real problems of society are compromised when the main actors of education, teachers, do not know or do not use effective methodologies and tools to match the mathematical knowledge of young people entering university.

The use of the technique called two-color Geometric Algebra is proposed as an alternative to equalize the knowledge of first semester students, knowing its effectiveness to reach the solution and understanding of exercises with basic Algebra polynomials, which are the basis of the knowledge of higher Algebra, thus relating it to what is established in the research line "Education and Pedagogy" to which the research project belongs FAPED, the faculty to which this master's degree is attached, promoting educational development, culture, The FAPED faculty, to which this master's degree is attached, promotes educational development, culture, ancestral knowledge and recreation in the Province of Esmeraldas, thus aiming to contribute to the fulfillment of the fourth sustainable development goal "Ensure inclusive, equitable and quality education and promote lifelong learning opportunities for all, intertwined with goal 1, 6 of the National Development Plan creating opportunities which "guarantees the right to health, education and integral care during the life cycle under quality criteria", landing this work in the fulfillment of the institutional objectives of UTELVT such as "e, h, i, and j". In this way, we intend to contribute to the solution of the low appropriation of basic concepts and poor understanding of the solution processes in algebraic subjects, which produces an inefficient mastery and low academic performance of students in the first levels of the careers in the administrative field of the Universidad Técnica Luis Vargas Torres.

An educational project presented by the government of Ecuador in 2007 indicated the following:

In recent years, and as a consequence of the great changes produced by the processes of globalization, the irruption of information and communication technologies and other phenomena and processes associated with the information society, there is an insistent questioning of what are the learning, the cultural knowledge, that have to form the frame of reference for the full exercise of citizenship. (Ministry of Popular Power for Education, 2007).

But the importance of mathematics was never doubted, since its use is fundamental in every field; in any type of business it is required to elaborate budgets, projections and analyze results; to model and build houses it is required to know the dimensions of a land; an anesthesiologist must know what dose to assign to the patient so that he can operate him; and this would not be possible without the application of the laws and procedures dictated by mathematics.

Understanding mathematics and its rules to complete the resolution processes that a problem requires is not easy for students, because most of them generally present an aversion or resistance to traditional teaching methods, considering the subject as a difficult and not very understandable subject. On the other hand, students who manage to understand the subject with ease lose interest in it, since they will have to wait until most of their classmates have understood the last topic for the teacher to move on to a new one.

The diagnostic test conducted in the academic period 2s-2021, applied to students of the Universidad Técnica Luis Vargas Torres de Esmeraldas who entered the first level of the career of Business Administration and Accounting and Auditing, yielded clear information, revealing a poor command of basic knowledge of mathematics. In order to advance in their higher studies, students must have a logical mathematical thinking and be able to solve problems with complex variables, therefore, it is essential to design

a methodological didactic proposal that ensures a rapid assimilation of the concepts and basic rules of mathematics for the resolution of algebraic problems. The non-use of creative techniques such as two-color geometric algebra causes a poor understanding of the basic algebraic fundamentals that serve as an anchor to consolidate higher mathematical knowledge and limit the academic progress of students who study majors related to mathematics.

Achieving the leveling of mathematical knowledge in a course is a complex task, since its fulfillment can be affected by different variables, among them we can indicate the individual desire to learn, the teacher's capacity to transmit his knowledge, the time available, the methodologies used, the maturity in the development of the student's autonomous work, the examples and exercises presented and the tasks assigned.

The mastery of the mathematical foundations and the resolution of problems with different unknowns, such as the majority of mathematical exercises proposed in the careers of business administration and accounting and auditing, is fundamental for the appropriation of concepts that serve as support in the mastery of higher knowledge, In the curricular mesh of the career many professionalizing subjects have as a prerequisite the approval of "Mathematics I" and "Mathematics II", sometimes the grades that students get in the exam to pass the subject do not reflect the real knowledge acquired, and this can be detrimental to the student, especially when he/she fails to understand all the topics.

Based on the search performed it can be indicated that in Ecuador at the moment there is only one research entitled <<Factorization with two-color Geometry>>, to deal with the same subject they use other names such as, for example; geometric algebra for polynomials or geometric algebra of Euclid, in addition there are many researches that aim to design a didactic proposal for the

teaching of algebra. This may be motivated by what Gyves, (2006) indicates in his research:

...in the last two decades the use of geometric methods has gradually increased (with the rise of computers and educational research). One of the reasons for this boom is the existence of studies in favor of the multiple use of representation registers. (p. 4).

He also mentions that "These studies explain that the plurality of semiotic systems -which consist of diverse representations of the same object- makes it possible to increase the cognitive capacity of the subjects" (Gyves, 2006).

The review of approximately forty publications has made it possible to delimit the problem and to evaluate research of greater relevance for the present work. Reviewed publications such as "Euclid's Geometric Algebra: an experience in the teaching of algebra", "Validation of the Delaunay triangulation using conformal geometric algebra", "Design of the didactic unit: Factorization with two-color geometry" and many more, demonstrate the great interest of the scientific community in the topic chosen for this work. All these researches are important because they allow us to know the results obtained with the application of algebra teaching methodologies with the representation of geometric figures and mark a starting point to achieve the objectives set for this research. The importance of the implementation of geometric figures for the explanation of algebra lies in the possibility of developing algebraic and geometric thinking, as indicated by Gómez (2021) in his research in which he states that geometric algebra "helps to develop two important thoughts for mathematics, the algebraic and the geometric" (p. 2). Villella (2001) mentions that the teaching of geometry involves offering elementary school students the possibility of describing, understanding and interpreting the world and its phenomena.

In the research conducted by Delgado and Butto (2015) one of the objectives is to investigate the feasibility of establishing a didactic sequence based on Euclid's geometric algebra and to investigate the algebraic representations of the relationship between different areas, for this, a qualitative methodology is used with the intention of understanding the meaning of the students' actions and to describe the phenomena that occur in the classroom, such as the difficulties presented by each student in the development of the proposed exercises. With this study, it is concluded that the didactic sequence for problem solving should start with the interconnection of the algebraic-geometric part from the algebraic knowledge and establish through geometry, the areas and their equation for rectangles and squares, with the purpose of interacting easily in the algebraic processes.

Guerra (2014) also presents a didactic proposal for factoring with two-color Geometry, in his final research work for obtaining the degree of Master in Education, In his work he uses the experimental method that allows him to observe, manipulate and record all the variables of study. For her proposal she considers necessary to develop 7 sessions, each one with a time of forty minutes, to deal with the topics presented:

Review and revision of contents. Remarkable products and remarkable quotients.

Common Factor and Common Factor by Grouping Terms. Trinomials.

Difference of perfect squares and Sum and difference of perfect cubes. Presentations and defense of the procedures of an exercise. Recovery with two-color Geometric Algebra (Guerra, 2014, p.18).

METHODOLOGY

In order to achieve each of the specific objectives of this study, it has been necessary to implement various research methods and an

instrument for data collection, which guarantees the reliability of the information.

A mixed methodology is used, which will allow the combination of qualitative and quantitative methods to achieve the objectives set out in this research.

For the development of this research, the problem was posed, a review of the state of the art was carried out, a hypothesis was formulated, data were collected from students, teachers and comparative analysis, applying an evaluation, an interview and a comparative table, finally, the data obtained were analyzed with which conclusions were drawn. With the comparative analysis of the two didactic proposals, the similarities and differences were determined, these were used to design and implement a new didactic proposal that was applied to the students of the university headquarters to verify the hypothesis raised. Therefore, we can indicate that the deductive method and the inductive method were applied.

Due to the nature of the present research work, the main method implemented is the Comparative Method, about this method Rus (2020) mentions that it is a way of elaborating or rejecting theory and hypotheses using comparisons based on procedures similar to scientific methods. So it is oriented to test the validity of an argument using science and the study of differences and similarities.

From the above we can affirm that the comparative method gives us the facility to determine, based on the study of the similarities and differences of a real argument, if a hypothesis or theory is approved or rejected by omitting a statistical procedure. In this case, thanks to the comparative method it will be possible to determine the differences between the didactic proposals reviewed and finally if the two-color geometric algebra improves the academic performance in mathematics of first semester

students of Business Administration and Accounting and Auditing of the Luis Vargas Torres Technical University of Esmeraldas.

Another indispensable methodology for the development of this research is the descriptive methodology, according to Cerda (1998, as cited in Bernal, 2010), "traditionally the word describe is defined as the act of representing, reproducing or depicting people, animals or things..."; and adds: "One must describe those most characteristic, distinctive and particular aspects of these people, situations or things, that is, those properties that make them recognizable to the eyes of others" (p.
71). Bernal (2010) indicates that "one of the main functions of descriptive research is the ability to select the fundamental characteristics of the object of study and its detailed description of the parts, categories or classes of that object" (p. 129).

The ethnographic technique and documentary analysis were essential for the process of obtaining and classifying the information, with the first one it was possible to obtain the necessary information to analyze the impact of two-color geometric algebra in the first semester courses of Business Administration and Accounting and Auditing of the Universidad Tecnica Luis Vargas Torres of Esmeraldas and with the bibliographic review it was possible to compare the revised methodological proposals.

RESULTS

One of the objectives of this research aims to identify the minimum knowledge of the population about two-color geometric algebra, to meet this objective it was essential to conduct an interview with 4 teachers of the Technical University Luis Vargas Torres of Esmeraldas who teach or have taught the subject Mathematics 1.

The first question addressed to the teachers was whether they had implemented the two-color geometric algebra didactic strategy for solving algebraic problems, the answer of all teachers was that they

had not used this strategy before, three teachers indicated that they had heard about this strategy, but did not know it in detail, one teacher said that he did not use it because there were no resources to apply this strategy.

The second question addressed to the teachers dealt with the methodologies used for teaching the operations of algebraic expressions. One teacher mentioned that he used the problem-based methodology, indicating that he posed problems with one or two unknowns, so that the student would first write down the linear equation that represents the problem, and then proceed to solve it. The other teachers stated that they use the classical methodology, which consists of explaining the concepts, presenting examples of solving the exercises, and proposing similar exercises.

In the third question, we asked about the difficulties that students have in the subject. All teachers agreed and stated that students have difficulties in solving exercises with rational whole and fractional algebraic expressions with a basic degree of complexity. In some cases, a minority of students also present difficulties in solving exercises due to the lack of knowledge of the rules of precedence of operations.

To the question "What are the most common errors in solving exercises? They responded that based on their experience, students do not manage to reach the solution of an exercise or problem because they do not apply a reflective strategy to reach the solution, so they make calculation errors, do not identify the available data, fail to identify the unknown, or do not complete the exercise.

In the penultimate question, "What are the topics planned in the syllabus and the expected achievements? The interviewees were able to state that in the first unit there is a review of the concepts, properties and relationships of natural numbers and integers

proper to arithmetic, and then they go on to deal with topics such as linear equations, fractional equations, equations with radicals, quadratic equations.

The last question of the survey was asked only to the teachers who currently teach Mathematics I. This was done after applying the didactic proposal, the question asked the teacher to indicate the benefits and results he/she could observe. The textual responses of the teachers are reported:

● Teacher 1

Students show greater interest in the classes, a greater predisposition to solve the proposed exercises is appreciated, and they manage to complete the exercises in less time.

● Teacher 2

With the implementation of geometric figures it is easier for students to solve the proposed exercises in a faster and safer way, greater enthusiasm is appreciated, the planned topics can be dealt with in less time, but it is necessary to have software that allows solving and saving the exercises.

CONCLUSIONS

The conclusions derived from the bibliographic analysis, the observation and verification of the research selected for analysis, and the results obtained from the surveys conducted with teachers and the evaluations taken from the students of Business Administration and Accounting and Auditing.

Based on the bibliographic review carried out to determine the minimum knowledge of the population on the topic: two-color geometric algebra, we can indicate that in Ecuador to date there is only one research that deals with the topic under this name, even so, we were able to determine that the topic of analysis is investigated only under the name of Geometric Algebra. There are numerous investigations that talk about the importance of

geometric algebra for and its applications in engineering, however, the investigations are limited in the analysis of the benefits of geometric algebra for the teaching of mathematics, it is highlighted that there is a strong separation of algebra and geometry in compulsory education. It has been observed that other Spanish-speaking countries have dealt with the subject and have elaborated didactic proposals that implement the geometric algebra strategy for teaching mathematics. In this study, the results and conclusions show that the implementation of a didactic proposal with the two-color geometric algebra strategy favors the teaching and learning process of algebraic problems and exercises.

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